CUWF Mathematical Conventions & Definitions

A Standard Mathematical Handbook for the CUWF Framework

Purpose. This handbook standardizes the mathematical notation, spaces, operators, tensors, and collapse-dynamical conventions used across the CUWF C-series. It is intended as a compact reference document for maintaining consistency across Paper C-2 and later mathematical extensions.

SECTION 1 — Manifolds and Spaces

1.1 Configuration Manifold $\mathcal{M}$

The configuration manifold $\mathcal{M}$ is the space of all collapse-node positions. Each point $x \in \mathcal{M}$ represents a possible geometric configuration of a collapse node.

$$x \in \mathcal{M}, \quad x = (x^1, x^2, \ldots, x^n)$$

The manifold dimension is denoted by n. The coordinates $x^1, x^2, \ldots, x^n$ provide a local coordinate system on $\mathcal{M}$.

1.2 Degree-of-Freedom Space M_DOF

The degree-of-freedom space M_DOF is the internal fiber space attached to every point $x \in \mathcal{M}$. It represents internal oscillation modes or hidden internal configuration variables associated with each collapse node.

$$q = (q^1, q^2, \ldots, q^K) \in M\_DOF$$

Here K denotes the number of internal DOF coordinates included in the chosen model.

1.3 Hybrid Product Manifold C × M_DOF

CUWF often works on a hybrid domain combining collapse-node configuration and internal DOF structure.

$$X = (x, q) \in C \times M\_DOF$$

C denotes the collapse-node configuration domain.

M_DOF denotes the internal DOF fiber.

X denotes the combined node state used in higher-level CUWF dynamics.

SECTION 2 — Metrics and Inner Products

2.1 Entropic Metric g_E

The entropic metric g_E is derived from the entropic field E(x). In the Hessian-based formulation, the metric evaluates the curvature response of E along tangent directions v and w.

$$g\_E(v, w) = v^{T} (\nabla^2 E)\, w$$

Equivalently, in component form, g_E is associated with the Hessian structure of E. This makes geometry an emergent property of the entropic field rather than an externally imposed spacetime metric.

2.2 Induced Inner Product

$$\langle v, w \rangle\_E = g\_E(v, w)$$

This inner product defines lengths, angles, and orthogonality relations inside the entropic geometry whenever g_E is positive-definite or locally nondegenerate.

2.3 Norm

$$\|v\|\_E = \mathrm{sqrt}(\langle v, v \rangle\_E)$$

The entropic norm measures the size of a vector with respect to the local curvature structure of E.

SECTION 3 — Operators

3.1 Gradient

$$\nabla E = (\partial E/\partial x^1, \ldots, \partial E/\partial x^n)$$

The gradient identifies the direction of steepest local change of the entropic field.

3.2 Divergence

$$\mathrm{div}(V) = \Sigma_i \, \partial V_i/\partial x_i$$

The divergence measures the net local expansion or contraction of a vector field V in configuration space.

3.3 Laplacian

$$\Delta E = \mathrm{div}(\nabla E)$$

The Laplacian measures scalar-field diffusion or smoothing. In CUWF, the standard Laplacian serves as a baseline before being generalized to entropic operators such as Δ_E.

3.4 Hessian

$$\mathrm{Hess}(E) = [\partial^2 E \,/\, \partial x_i \, \partial x_j]$$

The Hessian captures the second variation of E and provides the primary object from which entropic curvature and stability spectra are derived.

3.5 Linearized Operator

L_E_0 = Jacobian of collapse dynamics around equilibrium.

The linearized operator is used to analyze local stability around an equilibrium or fixed point of collapse dynamics.

SECTION 4 — Spectral Theory

4.1 Spectrum

$$\sigma(L) = \text{eigenvalues of } L$$

The spectrum records the eigenvalues of an operator L and is used to classify stability, instability, resonance, and transition behavior.

4.2 Pseudo-Spectrum

$$\sigma\_\varepsilon(L) = \{\lambda : \|(L - \lambda I)^{-1}\| > \varepsilon^{-1}\}$$

The pseudo-spectrum captures spectral sensitivity and near-instability. It is useful when non-normal operators or strongly coupled collapse dynamics amplify perturbations.

4.3 Resolvent

$$R(\lambda) = (L - \lambda I)^{-1}$$

The resolvent measures how strongly the system responds to spectral forcing near λ. Large resolvent norm indicates sensitivity and proximity to instability.

SECTION 5 — Stability and Curvature

5.1 Entropic Curvature

$$K\_E(v) = v^T \, \text{Hess}(E) \, v$$

Entropic curvature measures the directional second variation of E along v. Positive, zero, or negative values correspond to stable, marginal, or unstable geometric directions, respectively.

5.2 Stability Operator

$$\hat{S} = \text{Hess}(E)$$

The stability operator governs the local spectrum of collapse stability. Its eigenvalues classify stable modes, marginal modes, unstable directions, and tunnelling-ready configurations.

5.3 QC Boundary Condition

The quantum-classical transition occurs when the stability spectrum becomes weak enough that perturbative or noise-driven effects dominate.

$$\mathrm{Re}(\lambda) \longrightarrow 0 \quad \text{or} \quad \|R(\lambda)\| \longrightarrow \text{large}$$

In this regime, classical stability weakens and quantum-like behavior becomes accessible in CUWF terms.

SECTION 6 — Tensors

6.1 Entanglement Tensor

$$T_{ij} = \partial^2 E\_total / (\partial x_i \, \partial x_j)$$

The entanglement tensor captures cross-curvature coupling between nodes. Nonzero cross-blocks correspond to curvature inseparability, which is the CUWF geometric interpretation of entanglement.

6.2 Sensitivity Tensor (EST)

$$S\_E(\lambda) = R^\dagger R$$

The entropic sensitivity tensor measures spectral response intensity. It is especially useful for identifying regimes in which small perturbations are amplified by near-resonant collapse dynamics.

6.3 Multi-Node Block Operators

For N-node systems, the operator structure naturally forms block matrices with self-curvature blocks H_i and cross-curvature blocks C_{ij}.

H_block =

[H_1 C_{12} ⋯]

[C_{21} H_2 ⋯]

[⋱ ⋱ H_N]

The off-diagonal blocks encode multi-node coupling and entanglement-like curvature interaction.

SECTION 7 — Collapse Dynamics

7.1 Collapse Flow Equation

dx/dτ = −∇_E E(x)

The collapse-node trajectory follows the negative entropic gradient. Here τ denotes the entropic path parameter rather than ordinary physical time.

7.2 DOF Drift

dq/dτ = F(q; x)

Internal degrees of freedom evolve through a drift law coupled to the configuration position x. The specific functional form of F depends on the chosen CUWF model.

7.3 Multi-Node Collapse

dX/dτ = −∇_E E_total(X)

For multi-node systems, the combined node state X evolves along the negative entropic gradient of the total entropic field E_total.

SECTION 8 — Notation Table

Symbol	Meaning
$\mathcal{M}$	Configuration manifold
M_DOF	Internal DOF space
xi	Coordinate on $\mathcal{M}$
qj	DOF coordinate
E(x)	Entropic potential
g_E	Entropic metric
∇E	Gradient
ΔE	Laplacian
Hess(E)	Second variation of E
L_E0	Linear stability operator
σ(L)	Spectrum
σ_ε(L)	Pseudo-spectrum
R(λ)	Resolvent
K_E(v)	Entropic curvature
Ŝ	Stability operator
S_E(λ)	Sensitivity tensor EST
Tij	Entanglement tensor
H_block	Multi-node block operator
τ	Entropic time / entropic path parameter
X	Combined node state

Closing Note

This revised handbook preserves the original CUWF notation set while clarifying the role of each mathematical object. It can be used as a compact reference for Paper C-2 and as a standard convention document for later CUWF mathematical modules.