Appendices — Mathematical Notes and Operator-Stability Reference for Paper C-3

These appendices provide a compact mathematical reference for Paper C-3. They collect the operator notation, adjoint conventions, spectral definitions, resolvent geometry, stability-cone criteria, and multi-node stability structures used throughout the paper. They are intended as a reader-facing reference and do not replace the detailed derivations in the main text.

Appendix A — Operator Notation and Algebraic Conventions

Appendix A summarizes the primitive operator set and the algebraic conventions used in Paper C-3. The central feature of the CUWF operator system is that its operators are nonlinear, metric-dependent, and defined over the hybrid manifold C × M_DOF rather than over a fixed Hilbert space.

A.1 Primitive Operator Set

Symbol	Definition	Role in CUWF Operator Algebra
A	$A \equiv \Delta\_E$	Entropic Laplacian; curvature-smoothing sector.
B	$B \equiv \lvert\nabla E\rvert^2$	Nonlinear gradient-squared sector; slope sharpening and collapse-wall formation.

C	$C \equiv D\_DOF$	DOF-manifold derivative; fiber drift and internal-degree deformation.
$\mathcal{G}$	$\mathcal{G} = \{A, B, C\}$	Minimal generating set of the CUWF operator algebra.
$\mathfrak{E}\_E$	$\mathfrak{E}\_E = A - \alpha B + \beta C$	CUWF Master Operator governing entropic operator flow.

A.2 Operator Composition

$$(X \circ Y)(f) = X(Y(f))$$

Composition is associative at the formal level but generally non-distributive because A and B depend on the entropic metric g_E, derivatives of E, and nonlinear gradient structure. Consequently, the CUWF algebra is not a Lie algebra and not a C*-algebra.

$$X \circ (Y \circ Z) = (X \circ Y) \circ Z$$

$$A \circ (B + C) \neq A \circ B + A \circ C$$

A.3 Commutator and Anti-Commutator

$$[X,Y]_* = X \circ Y - Y \circ X$$

$$\{X,Y\} = X \circ Y + Y \circ X$$

• $[A,B]_* \neq 0$: curvature diffusion and slope sharpening do not commute.

• $[A,C]_* \neq 0$: configuration-space curvature and DOF-fiber drift do not commute.

• $[B,C]_* \neq 0$: gradient sharpening depends on DOF deformation.

• $[\mathfrak{E}_E,A]_*$, $[\mathfrak{E}_E,B]_*$, and $[\mathfrak{E}_E,C]_*$ generate operator evolution and spectral deformation.

A.4 Composite Curvature Operator

$$K_E = A^2 - A \circ B + B \circ A$$

K_E collects the higher-order interaction between entropic curvature flow and nonlinear gradient sharpening. It is useful when describing rigidity, collapse-wall formation, and higher-order curvature response.

A.5 Nonlinear Entropic Operator Tower (NEOT)

$$T_0 = I$$

$$T_1 = A - \alpha B + \beta C = \mathfrak{E}_E$$

$$T_{n+1} = A(T_n) - \alpha B(T_n) + \beta C(T_n)$$

NEOT describes repeated nonlinear application of the CUWF Master Operator. It is the natural tower of higher-order entropic evolution terms and provides the basis for future nonlinear quantization and higher-order stability analysis.

Appendix B — Adjoint and Inner Product Conventions

Appendix B collects the inner-product and adjoint conventions needed for Sections O-3 through O-6. Because g_E depends on E, all adjoints in CUWF are state-dependent and can change as the entropic field evolves.

B.1 Entropic-Metric Inner Product

$$\langle f,g\rangle_{g_E} = \int f\, g \sqrt{|g_E|}\, dx$$

When DOF-fiber coordinates are included, the measure may be extended to the hybrid manifold C × M_{DOF}:

$$\langle f,g\rangle_{g_E} = \int_C \int_{M_{DOF}} f\, g \sqrt{|g_E|} \sqrt{|h|}\, dx\, dq$$

B.2 Definition of the Adjoint

$$\langle Xf,g\rangle_{g_E} = \langle f,X^\dagger g\rangle_{g_E}$$

The adjoint $X^\dagger$ is not fixed once and for all. It depends on the entropic metric, and the metric depends on E, ∇E, and Hess(E).

B.3 Adjoint of Primitive Operators

Operator	Adjoint Relation	Meaning
$A = \Delta\_E$	$A^\dagger = A$	Curvature diffusion is self-adjoint under the entropic metric.
$B = \|\nabla E\|^2$	$B^\dagger = B + Q\_B$	Slope sharpening gains metric-variation correction terms.
$C = D\_DOF$	$C^\dagger = -C + \Gamma\_DOF$	DOF-fiber derivative is anti-Hermitian only in the flat-connection limit.
$\mathfrak{E}\_E$	$\mathfrak{E}\_E^\dagger = A - \alpha(B + Q\_B) + \beta(-C + \Gamma\_DOF)$	The Master Operator is intrinsically mixed-adjoint.

B.4 Non-Normality

$$L L^\dagger \neq L^\dagger L$$

Non-normality is central to Paper C-3. It allows transient amplification, pseudo-spectral expansion, metastability, tunneling readiness, and quantum-like activation even when the ordinary spectrum appears stable.

B.5 Multi-Node Adjoint Blocks

$$A_{ij}^\dagger = A_{ji}$$

Cross-curvature blocks can be adjoint-symmetric even when the full operator algebra remains non-Hermitian. This property underlies deterministic entanglement as cross-node curvature alignment.

Appendix C — Spectrum, Pseudo-Spectrum, and Resolvent Reference

Appendix C summarizes the spectral tools used in Paper C-3. CUWF requires nonlinear spectral theory because the operator algebra is nonlinear, metric-dependent, and non-normal.

C.1 Nonlinear Entropic Spectrum

$$X(f) = \lambda(f)\, f$$

$$\sigma_E(X) = \{ \lambda(f) : X(f) = \lambda(f)\, f \}$$

The entropic eigenvalue $\lambda(f)$ is a functional rather than a fixed scalar. It depends on the entropic field, the metric, and the eigenfunction itself.

C.2 CUWF Master-Operator Eigenvalue Relation

$$\mathfrak{E}_E(f) = \lambda(f)\, f$$

$$A(f) - \alpha|\nabla E|^2 f + \beta D_{DOF}(f) = \lambda(f)\, f$$

A Rayleigh-like expression for $\lambda(f)$ may be written using the entropic inner product:

$$\lambda(f) = \langle A(f),f\rangle/\langle f,f\rangle - \alpha\langle|\nabla E|^2 f,f\rangle/\langle f,f\rangle + \beta\langle D\_DOF(f),f\rangle/\langle f,f\rangle$$

C.3 Linearized Operator

$$L_{E0} = (d/d\varepsilon)|_{\varepsilon=0}\, \mathfrak{E}\_E[E_0 + \varepsilon\delta E]$$

L_{E0} describes the first-order response of the CUWF Master Operator near a background entropic field E_0. It is the operator used in pseudo-spectral stability analysis.

C.4 CUWF Pseudo-Spectrum

$$\sigma\_\varepsilon(L_{E0}) = \{ \lambda \in C : \|(L_{E0} - \lambda I)^{-1}\|_{g_E} > \varepsilon^{-1} \}$$

$$\lambda \in \sigma\_\varepsilon(L_{E0}) \Leftrightarrow \|(L_{E0} - \lambda I)f\|_{g_E} < \varepsilon,\ \|f\|_{g_E}=1$$

The pseudo-spectrum detects hidden instability and non-normal amplification that the ordinary spectrum cannot capture.

C.5 Resolvent

$$R(\lambda;E_0) = (L_{E0} - \lambda I)^{-1}$$

$$\partial\sigma\_\varepsilon(L_{E0}) : \|R(\lambda;E_0)\|_{g_E} = \varepsilon^{-1}$$

The resolvent measures how strongly a perturbation is amplified near λ. Large resolvent norm indicates near-singularity, metastability, or tunneling readiness.

C.6 Spectral Flow

$$d\lambda/d\tau = -\langle\delta\lambda/\delta E, \mathfrak{E}\_E(E)\rangle + \langle\delta\lambda/\delta f, \partial f/\partial\tau\rangle$$

This expression describes how the entropic spectrum changes as the field evolves under CUWF flow.

Appendix D — Entropic Sensitivity Tensor and Stability Cone

Appendix D collects the directional geometric objects used to translate pseudo-spectrum and resolvent growth into concrete tunneling paths, metastability directions, and quantum–classical boundary conditions.

D.1 Entropic Sensitivity Tensor (EST)

$$S_{E0}(\lambda) = R(\lambda;E_0)^\dagger R(\lambda;E_0)$$

$S_{E0}(\lambda)$ is positive semi-definite. Its eigenvalues give squared amplification factors, and its eigenvectors identify the soft directions in entropic geometry.

D.2 Dominant Soft Direction

$$\mu_{max}(\lambda) = \sup_{u : \|u\|=1} \|R(\lambda)u\|^2_{g_E}$$

$$S_{E0}(\lambda)\, v_{soft} = \mu_{max}\, v_{soft}$$

The vector v_{soft} is the dominant soft direction. In CUWF, tunneling corresponds to motion along this direction when the pseudo-spectrum reaches the quantum-active boundary.

D.3 EST Block Decomposition

$$S = [\, S_{xx} \quad S_{xq} \,;\, S_{qx} \quad S_{qq} \,]$$

- S_{xx} describes soft modes in configuration space.
- S_{qq} describes DOF-drift sensitivity.
- S_{xq} and S_{qx} describe cross-coupling channels.
- Entanglement onset occurs when S_{xq} becomes comparable in magnitude to S_{xx}.

D.4 Directional Entropic Curvature

$$K\_E(v) = v^{T}(\nabla^2 E)v$$

Condition	CUWF Interpretation	Regime
K_E(v) > 0	Locally convex entropic curvature	Classical / stable
K_E(v) = 0	Flat soft direction	Metastable / marginal
K_E(v) < 0	Local concavity	Quantum-active / tunneling-ready

D.5 Stability, Soft, and Quantum Cones

$$C\_stable(E_0) = \{ v : K\_E(v) > 0 \}$$

$$C\_soft(E_0) = \{ v : K\_E(v) = 0 \}$$

$$C\_quantum(E_0) = \{ v : K\_E(v) < 0 \}$$

These cones partition the tangent space of the entropic manifold into stable, marginal, and quantum-active directions.

D.6 Quantum–Classical Boundary

$$\text{Classical regime} \Leftrightarrow \mathrm{Re}(\sigma\_\varepsilon(L\_\{E0\})) < 0$$

$$\text{Quantum regime} \Leftrightarrow \exists\, \tilde{\lambda} \in \sigma\_\varepsilon(L\_\{E0\}) \text{ such that } \mathrm{Re}(\tilde{\lambda}) \geq 0$$

$$\partial QC(E_0) = \{ \lambda \in C : \|(L\_\{E0\} - \lambda I)\text{^}\{-1\}\|\_\{g\_E\} = \varepsilon\text{^}\{-1\} \}$$

The quantum–classical boundary is a geometric boundary in operator space, not an energy threshold or probabilistic postulate.

Appendix E — Multi-Node Stability and Collective Modes

Appendix E summarizes the multi-node extension of Paper C-3. In CUWF, entanglement and collective tunneling arise from block-operator geometry rather than Hilbert-space tensor products.

E.1 Multi-Node Hybrid Manifold

$$\mathcal{M}_{\mathbf{n}} = (C \times M\_DOF)^{\mathbf{n}}$$

Each collapse-node perturbs the entropic metric and contributes to the global curvature structure. Therefore, stability is a network property rather than a purely local property.

E.2 Block Hessian

$$H(E) = [\ H_{11}\ \ H_{12}\ \cdots\ H_{1n}\ ;\ H_{21}\ \ H_{22}\ \cdots\ H_{2n}\ ;\ \vdots\ \ \vdots\ \ \ddots\ \ \vdots\ ;\ H_{n1}\ \ H_{n2}\ \cdots\ H_{nn}\ ]$$

• H_ii describes on-node curvature, slope terms, and DOF internal Hessian.

• H_ij for i $\neq$ j describes cross-node coupling curvature, DOF-mediated interaction, and metric-induced deformation.

• H_ij $\neq$ 0 indicates that node i and node j participate in shared entropic curvature.

E.3 Multi-Node Pseudo-Spectrum

$$\sigma\_\varepsilon(\mathcal{L}) = \{\ \lambda : \|(\mathcal{L} - \lambda I)\text{^\{-1\}}\|\_\{g\_E\} > \varepsilon\text{^\{-1\}}\ \}$$

The multi-node pseudo-spectrum is generally larger than the union of single-node pseudo-spectra because coupling expands the resolvent norm and propagates soft directions across the network.

E.4 Deterministic Entanglement Mode

$$H\ v\_ent = \lambda\_soft\ v\_ent, \quad v\_ent \neq \text{localized}$$

A deterministic entanglement mode is a collective soft eigenmode with non-zero support across multiple nodes. No Hilbert-space tensor product or measurement postulate is required.

E.5 Collective Tunneling Criterion

$$\lambda\_min(H) = 0$$

$$\min\_\{v \in T\mathcal{M}_n,\ \|v\|=1\}\ v^T H v = 0$$

Collective tunneling activates when the collective curvature minimum becomes zero or negative. This opens a network-wide tunneling corridor and synchronizes collapse-node drift.

Appendix F — Quick Reference Table of CUWF Stability Regimes

Appendix F provides a compact reference table for the stability regimes used throughout Paper C-3.

Regime	Mathematical Condition	CUWF Meaning	Primary Diagnostic
Classical stable	Re(σ_ε(L_E0)) < 0 and K_E(v) > 0 for all relevant v	Entropic convexity; stable collapse-node flow	Ordinary spectrum and stability cone
Metastable	Re(σ_ε(L_E0)) ≈ 0 or K_E(v) = 0	Soft plateau; delayed transition; near-critical resolvent growth	Pseudo-spectrum boundary and EST

Quantum-active	$\exists \tilde{\lambda} \in \sigma\_\varepsilon(L\_E0)$, $Re(\tilde{\lambda}) \geq 0$	Pseudo-spectral activation of a soft direction	Resolvent growth and v_soft
Tunneling-ready	min K_E(v) = 0	Threshold of basin escape	Stability cone boundary
Tunneling activated	min K_E(v) < 0 or $\mu\_max \geq \varepsilon^{-2}$	Deterministic transition through a soft entropic corridor	EST and pseudo-spectrum crossing
Entangled	Cross-node soft modes align; H_ij $\neq$ 0; v_ent non-localized	Collective geometric correlation across nodes	Block Hessian and multi-node pseudo-spectrum
Collective instability	$Re(\sigma\_\varepsilon(\mathcal{L}\_E0)) \geq 0$	Network-wide activation or synchronized collapse	Block resolvent

Final Note on the Role of These Appendices

The appendices are intentionally compact. Their purpose is to give readers a fast reference for the mathematical objects that recur throughout Paper C-3. Detailed derivations remain in the corresponding main sections O-0 through O-6, while these appendices serve as a stable notation and interpretation layer for future C-series papers.