Section O-1 — Algebraic Structure of CUWF Operators

Section O-1 introduces the algebraic backbone of the CUWF operator system. Unlike linear Hilbert-space operator algebras, CUWF operators are nonlinear, metric-dependent, defined on hybrid ($C \times M_{DOF}$) manifolds, non-Hermitian under standard inner products, and mixed-adjoint under the entropic metric inner product $\langle \cdot,\cdot \rangle_{g_E}$.

This section defines the CUWF operator set, their algebraic relations, commutators and anti-commutators, adjoint structures under g_E, and the Nonlinear CUWF Operator Algebra (NCOA), which becomes the core algebraic object of Paper C-3.

O-1.1 CUWF Operator Set

Let the primary operators be:

$$A \equiv \Delta_E$$

where A is the entropic Laplacian;

$$B \equiv |\nabla E|^2$$

where B is the nonlinear gradient-squared operator; and

$$C \equiv D_{DOF}$$

where C is the DOF-manifold derivative.

These form the minimal generating set:

$$\mathcal{G} = \{ A, B, C \}$$

The CUWF Master Operator is: $\mathcal{E}\_E = A - \alpha B + \beta C$

This expression identifies the three irreducible operator channels through which CUWF dynamics are generated: entropic smoothing, nonlinear slope sharpening, and DOF-fiber deformation.

O-1.2 Linear vs Nonlinear Operator Structure

Unlike conventional operator algebras:

A is linear in the argument f.

B is nonlinear: $B(f) = |\nabla E|^2 f$.

C is linear but domain-restricted to DOF dimensions.

$\mathcal{E}\_E$ is nonlinear because B appears multiplicatively.

Thus:

$$\mathcal{E}\_E(f + g) \neq \mathcal{E}\_E(f) + \mathcal{E}\_E(g)$$

Nonlinearity is intrinsic to CUWF; it is not a perturbation added after a linear theory has already been assumed.

O-1.3 Operator Composition Rules

Define the composition of operators X and Y by:

$$(X \circ Y)(f) = X(Y(f))$$

Because the metric g_E depends on E(x, DOF), compositions involving A or B generally depend on ∇E, Hessian(E), and derivatives of g_E.

The key results are:

$A \circ A$ is a well-defined elliptic composition.

$A \circ B$ introduces third derivatives of E.

$B \circ A$ introduces multiplicative curvature–gradient coupling.

$B \circ B$ is nonlinear and not reducible.

$C \circ A \neq A \circ C$ because DOF derivatives and spatial derivatives do not commute.

$C \circ B$ is well-defined but nonlinear in E.

These rules show that CUWF operator composition is algebraically meaningful but geometrically nontrivial, because the operators act on a metric that is itself generated by the evolving entropic field.

O-1.4 Commutator Algebra

Define the CUWF commutator:

$$[X, Y] = X \circ Y - Y \circ X$$

General properties:

$[A, B] \neq 0$

$[A, C] \neq 0$

$[B, C] \neq 0$

$[\mathcal{E}_E, B] \neq 0$

$[\mathcal{E}_E, A] \neq 0$

This distinguishes CUWF from GR, where commutators are not emphasized, and from QM/QFT, where commutators define the algebra but the operators are linear.

O-1.4.1 Commutator [A, B]

$$[A, B]f = A(|\nabla E|^2 f) - |\nabla E|^2 A(f)$$

This produces $\partial(|\nabla E|^2)/\partial x$ terms, curvature–gradient coupling, and third-order geometric derivatives. Thus [A, B] encodes higher-order entropic geometry.

O-1.4.2 Commutator [A, C]

$$[A, C]f = A(D\_DOF\ f) - D\_DOF(Af)$$

The operators do not commute because A differentiates with respect to configuration coordinates, while C differentiates along the DOF fiber. The cross-term corresponds to entropic gauge-like effects in later sections.

O-1.4.3 Commutator [B, C]

$$[B, C]f = |\nabla E|^2\ D\_DOF(f) - D\_DOF(|\nabla E|^2\ f)$$

This encodes the DOF-dependence of $|\nabla E|^2$ and therefore describes how internal DOF structure modifies entropic-gradient sharpening.

O-1.5 Anti-Commutator Algebra

Define:

$$\{X, Y\} = X \circ Y + Y \circ X$$

CUWF requires the anti-commutator for spectral analysis and variational derivations.

Key structure:

{A, B} is nonlinear symmetric composition.
{A, C} mixes DOF and geometric curvature terms.
{B, C} is a multiplicative–derivative symmetric operator.

Together with commutators, anti-commutators provide the symmetric operator combinations required for stability and second-variation analysis.

O-1.6 Adjoint Operators Under g_E Metric

Define the adjoint $X^\dagger$ such that:

$$\langle Xf, g\rangle_{g_E} = \langle f, X^\dagger g\rangle_{g_E}$$

The principal adjoint results are:

$A = \Delta_E$ is self-adjoint: $A^\dagger = A$

B is not self-adjoint: $B^\dagger \neq B$

because g_E depends on E and B depends on ∇E.

C is generally non-self-adjoint: $C^\dagger \neq C$

unless M_{DOF} has a symmetric connection.

Thus $\mathcal{E}_E$ is mixed-adjoint: $\mathcal{E}_E^\dagger = A + \alpha B^\dagger + \beta C^\dagger$

$$\mathcal{E}_E^\dagger \neq \mathcal{E}_E$$

This mixed-adjoint structure is a central reason why CUWF evolution is not unitary, not Hilbert-linear, and not reducible to conventional quantum operator evolution.

O-1.7 Nonlinear CUWF Operator Algebra (NCOA)

We define the CUWF operator algebra:

$$\mathcal{A}_{CUWF} = \langle\, A, B, C \mid \text{composition, commutators, anti-commutators} \,\rangle$$

This structure is nonlinear, metric-dependent, defined on hybrid manifolds, non-Hermitian, fundamentally different from C* or Lie algebras, and compatible with variational and spectral theory.

NCOA is the core object of Paper C-3 and the basis for the development from O-2 through O-6.

O-1.8 Purpose of O-1

This section:

defines the operator generating set;

shows nonlinearity as fundamental;

derives algebraic relations;

introduces commutators and adjoints;

defines the CUWF operator algebra $\mathcal{A}$_CUWF.

O-1 completes the algebraic foundation that all later operator theory—spectral, adjoint, evolutionary, and quantization-oriented—will build on.