Section O-6 — Stability Operators and Spectral Criteria

Section O-6 develops the complete stability theory required for CUWF operator dynamics. While Sections O-3 to O-5 focused on adjoint structure, nonlinear spectrum, and operator evolution, the present section isolates the pure stability sector encoded in the second variation of the entropic field. It then extends that sector into spectral classes, pseudo-spectral stability, quantum-classical boundary conditions, non-normal growth, resolvent geometry, stability cones, and multi-node collective eigenmodes.

The central object is the CUWF stability operator Ŝ, defined on the hybrid manifold H = C × M_DOF and interpreted through the entropic metric g_E. This section shows how classical stability, metastability, tunneling, entanglement, and collective collapse can all be represented as different geometric regimes of the same operator framework.

O-6.1 Stability Operator Ŝ = Hess(E) on the Hybrid Manifold

Section O-6.1 introduces the CUWF stability operator Ŝ, defined as the second-variation operator, or Hessian, of the entropic field E on the hybrid manifold H = C × M_DOF. While Sections O-3–O-5 focused on operator evolution and nonlinear spectrum of the Master Operator $\mathcal{E}$_E, here we isolate the pure stability sector encoded in Hess(E) and express it in a form consistent with the CUWF operator algebra.

The goals of O-6.1 are to:

define the second-variation operator Ŝ on C × M_DOF;

express Ŝ as a metric-weighted Hessian with g_E;

decompose Ŝ into curvature, slope, and DOF contributions;

separate self-adjoint and adjoint-asymmetric parts;

relate $\hat{S}$ to the linearized operator L_{E0} of $\mathcal{E}$_E.

O-6.1.1 Second-Variation Operator on C × M_DOF

Consider small perturbations δE around a reference entropic field E_0 on the hybrid manifold H = C × M_DOF. The second variation of E along a perturbation δE is defined formally by

$$\delta^2 E \equiv \langle \delta E , \hat{S}[\, E_0 \,]\, \delta E \rangle\_\{g\_E\},$$

where $\hat{S}[\, E_0 \,]$ is the CUWF stability operator evaluated at the background configuration E_0. By definition, $\hat{S}[\, E_0 \,]$ is the Hessian of E as a functional on the space of admissible fields:

$$\hat{S}[\, E_0 \,] = \mathrm{Hess\_E(E)|\_\{E} = E_0\}.$$

Acting on a perturbation δE, $\hat{S}$ produces the second-order restoring or destabilizing response of the entropic geometry to that perturbation. Thus:

$\hat{S}$ encodes local convexity or concavity of the entropic landscape;

positive directions of $\hat{S}$ correspond to stabilizing curvature;

negative directions correspond to instability or tunneling directions;

zero modes correspond to marginally flat directions in entropic geometry.

O-6.1.2 Metric-Weighted Hessian with g_E

Because CUWF is formulated with the entropic metric g_E, the Hessian cannot be taken with respect to a flat background. Instead, the stability operator is a metric-weighted Hessian. On the configuration manifold C with coordinates x^i and on the DOF manifold M_DOF with coordinates q^a, we write schematically

$$\hat{S}[\, E_0 \,]\delta E = (\nabla\_i\nabla\_j\ E_0)\ g\_E\verb|^|\{ij\}\ \delta E + (\nabla\_a\nabla\_b\ E_0)\ h\verb|^|\{ab\}\ \delta E + \text{mixed terms},$$

where ∇_i is the g_E-compatible covariant derivative on C, ∇_a is the DOF-fiber covariant derivative on M_DOF, h^{ab} denotes the effective DOF metric induced by the DOF connection, and the mixed terms couple configuration and DOF directions.

The inner product entering the second variation is always

$$\langle \delta E_1 , \delta E_2 \rangle_{g_E} = \int_C \int_{M_{DOF}} \delta E_1 \, \delta E_2 \sqrt{|g_E|} \sqrt{|h|} \, dx \, dq.$$

Thus $\hat{S}$ is not a bare Hessian; it is the Hessian expressed in the entropic metric measure. This guarantees compatibility with the adjoint relations of Section O-3, the spectrum definitions of Section O-4, and the evolution equations of Section O-5.

O-6.1.3 Decomposition into Curvature, Slope, and DOF Parts

The CUWF Master Operator $\mathcal{E}_E = A - \alpha B + \beta C$ naturally suggests a decomposition of $\hat{S}$ into three sectors:

$$\hat{S} = \hat{S}_{curv} - \alpha \, \hat{S}_{slope} + \beta \, \hat{S}_{DOF}.$$

The curvature sector $\hat{S}_{curv}$ is the pure Laplacian part of the Hessian:

$$\hat{S}_{curv}[E_0]\delta E \approx \text{(second variation of } A(E))|_{E_0} \text{ acting on } \delta E.$$

Geometrically, this sector measures how Laplacian curvature responds to changes in E, encodes stabilizing diffusive effects of entropic curvature, and typically yields self-adjoint elliptic contributions under $\langle \cdot,\cdot \rangle_{g_E}$.

The slope sector $\hat{S}_{slope}$ arises from the $B = |\nabla E|^2$ contribution:

$$\hat{S}_{slope}[E_0]\delta E \approx \text{(second variation of } |\nabla E|^2)|_{E_0} \text{ acting on } \delta E.$$

It contains terms proportional to $\nabla E_0 \cdot \nabla(\delta E)$, quadratic gradient couplings in δE, and directions in which slope sharpening reinforces or opposes curvature diffusion. This sector is generically non-self-adjoint and can flip sign depending on the structure of ∇E_0, thereby producing stability, instability, or tunneling directions.

The DOF sector $\hat{S}_{DOF}$ derives from the DOF derivative $C = D_{DOF}$:

$$\hat{S}\_DOF[E_0]\delta E \approx \text{(second variation of D\_DOF E)}|\_\{E_0\} \text{ acting on } \delta E.$$

It encodes curvature along DOF fibers, twist or shear of entropic geometry across internal degrees of freedom, and contributions from the DOF connection and its curvature. This sector is responsible for DOF-induced instability directions, DOF-driven tunneling channels, and entanglement-mode deformation in multi-node geometries.

O-6.1.4 Self-Adjoint Sector vs Adjoint-Asymmetric Sector

With respect to the entropic inner product $\langle\cdot,\cdot\rangle$_{g_E}, the stability operator $\hat{S}$ splits naturally into symmetric and antisymmetric parts:

$$\hat{S}\_sym = (\hat{S} + \hat{S}\dagger) / 2$$

$$\hat{S}\_asym = (\hat{S} - \hat{S}\dagger) / 2.$$

The symmetric part $\hat{S}$_sym controls purely real stability: positive, negative, and zero modes in the sense of classical second-variation analysis. The adjoint-asymmetric part $\hat{S}$_asym does not change the real stability index directly, but it generates skew-flow components crucial for tunneling and non-normal amplification.

In CUWF terms, $\hat{S}$_curv contributes dominantly to $\hat{S}$_sym, $\hat{S}$_slope contributes to both $\hat{S}$_sym and $\hat{S}$_asym depending on ∇E_0 and metric variations, and $\hat{S}$_DOF contributes strongly to $\hat{S}$_asym through the DOF connection. Thus the decomposition $\hat{S}$ = $\hat{S}$_sym + $\hat{S}$_asym forms the operator-theoretic bridge between classical stability analysis and CUWF tunneling or non-normal behavior.

O-6.1.5 Relation to the Linearized Operator L_{E0}

Let $\mathcal{E}$_E be the CUWF Master Operator and L_{E0} its linearization around E_0:

$$L\_\{E0\}\delta E \equiv (d/d\varepsilon)|\_\{\varepsilon=0\}\ \mathcal{E}\_E[E_0 + \varepsilon\delta E].$$

In Section O-4, L_{E0} was used to define the linearized spectrum and pseudo-spectrum. The stability operator $\hat{S}[E_0]$ is related but conceptually distinct. L_{E0} encodes how the evolution operator $\mathcal{E}$_E changes to first order with δE, whereas $\hat{S}[E_0]$ encodes the second variation, or curvature, of the entropic functional itself along δE.

Formally, the two are linked through

$$\delta^2 E \approx \langle \delta E , \hat{S}[E_0]\delta E\rangle_{g\_E}$$

and

$$\partial E/\partial \tau = -\mathcal{E}\_E(E).$$

Therefore, the sign structure of $\hat{S}[E_0]$ constrains which eigenmodes of L_{E0} can become unstable. Directions where $\hat{S}$ has negative eigenvalues correspond to potential tunneling or instability modes in the full nonlinear dynamics. The self-adjoint part of L_{E0} is controlled by $\hat{S}$_sym, while its non-normal, adjoint-asymmetric behavior is constrained by $\hat{S}$_asym. In this sense, $\hat{S}[E_0]$ provides the static stability landscape of entropic geometry, whereas L_{E0} provides the dynamic linearized flow directions on that landscape.

O-6.2 Spectral Classes of the CUWF Stability Operator $\hat{S}$

Section O-6.2 classifies the eigenmodes of the stability operator $\hat{S}$ = Hess(E), understood as the metric-weighted hybrid-manifold Hessian, into distinct spectral classes. Each class has a precise meaning in CUWF entropic geometry and a direct physical interpretation in CUWF cosmology, tunneling, and entanglement.

The operator $\hat{S}$ acts on perturbations δE through

$$\hat{S}[E_0]\delta E = \lambda \delta E,$$

where $\lambda \in \mathbb{R}$ for symmetric directions and $\lambda \in \mathbb{C}$ for adjoint-asymmetric directions. Because $\hat{S}$ is metric-weighted, non-flat, and partially non-self-adjoint, eigenvalues may be real or complex, zero modes require special treatment, negative modes define tunneling channels, and complex modes define non-normal geometric drift. We divide the $\hat{S}$-spectrum into five primary CUWF spectral classes.

O-6.2.1 Positive Modes (Stabilizing Curvature Directions)

Positive eigenvalues satisfy

$$\lambda > 0.$$

In these modes, δE is restored toward E_0, local entropic curvature is convex, collapse geometry is locally stable, perturbations decay under the flow, and the system is associated with curvature-dominant basins. Mathematically, $\delta^2 E > 0$ along these modes, giving strictly positive second variation. Physically, positive modes define the stability of collapse minima, correspond to classical non-quantum behavior, and contribute to the classical regimes discussed in O-6.3.

O-6.2.2 Zero Modes (Marginal Directions / Flat Entropic Geometry)

Zero modes satisfy

$$\lambda = 0.$$

They indicate that δE lies along a direction of marginal curvature, entropic flatness, or symmetry-like degeneracy. Geometrically, these are pathways with no restoring force; a collapse node can drift freely along them. They often mark precursor regions for bifurcation in O-4 and correspond to entropic ridges or flat saddles. Zero modes are crucial for the emergence of soft modes in CUWF cosmology, the onset of multi-node entanglement, and the transition from classical to quantum regimes.

O-6.2.3 Negative Modes (Instability / Tunneling Directions)

Negative eigenvalues satisfy

$$\lambda < 0.$$

Negative modes correspond to directions where $\delta^2 E < 0$. Perturbations amplify, E_0 becomes unstable along δE, entropic geometry curves downward, collapse does not remain in the basin, and tunneling channels open. Physically, these directions define CUWF tunneling, transitions between entropic basins, pre-collapse instability, and entropic saddle regions. This is the operator-theoretic foundation for the tunneling phenomena discussed in Paper A-4 and the cosmic phase-transition mechanisms in Papers A-5 and A-6.

O-6.2.4 Complex Modes (Adjoint-Asymmetric Directions)

Complex modes arise when $\hat{S}_{asym} \neq 0$:

$$\lambda = \lambda_R \pm i\lambda_I, \text{ with } \lambda_I \neq 0.$$

They arise from DOF-connection curvature, slope-sector adjoint-asymmetry, and variation of the metric g_E. The real part λ_R determines growth or decay, while the imaginary part λ_I generates rotation or drift in function space. These modes reflect non-normal, non-Hermitian structure and provide the basis for geometric tunneling even when $\lambda_R > 0$. Complex modes indicate possible oscillatory collapse, tunneling without classical instability, and entanglement deformation between nodes on M_{DOF}. They are not possible in classical Hessian theory and arise only because CUWF is defined on a hybrid manifold with adjoint-asymmetric operators.

O-6.2.5 Pseudo-Spectral Modes (Non-Normal Amplification)

Even when true eigenvalues λ satisfy

$$\mathrm{Re}(\lambda) < 0,$$

the pseudo-spectrum may contain $\tilde{\lambda}$ with

$$\mathrm{Re}(\tilde{\lambda}) \geq 0.$$

This occurs when the operator is strongly non-normal. Small perturbations can blow up, transient growth can dominate, changes in g_E can amplify responses, and DOF drift can generate near-resonant modes. In CUWF, pseudo-spectral modes signal metastability, define pre-instability thresholds, identify hidden entropic channels through which collapse may escape, and predict quantum-classical boundary shifts.

O-6.2.6 Summary Table of $\hat{S}$ Spectral Classes

Spectral Class	Condition	Meaning in CUWF Geometry	Physical Interpretation
Positive modes	$\lambda > 0$	convex curvature	stable / classical regime
Zero modes	$\lambda = 0$	flat entropic geometry	symmetry, marginal stability
Negative modes	$\lambda < 0$	concave curvature	instability / tunneling
Complex modes	$\lambda = \lambda_R \pm i\lambda_I$	adjoint-asymmetry drift	oscillatory / entanglement deformation
Pseudo-spectral modes	$\tilde{\lambda}$ from σ_ε	non-normal amplification	metastability / hidden channels

O-6.3 CUWF Pseudo-Spectral Stability

Section O-6.3 establishes the pseudo-spectral stability framework for CUWF, extending classical spectral analysis to the fully non-normal, metric-dependent linearized operator

$$L_{E0} = (d/d\varepsilon)|_{\varepsilon=0}\, \mathcal{E}_E[E_0 + \varepsilon\delta E].$$

Because CUWF operators are non-self-adjoint, metric-dependent, defined on the hybrid manifold $C \times M_{DOF}$, and perturbed by curvature, slope, and DOF connection, the classical spectrum is insufficient for determining stability. CUWF therefore adopts a pseudo-spectral approach based on σ_ε.

O-6.3.1 Definition of CUWF Pseudo-Spectrum $\sigma_\varepsilon(L_{E0})$

Let the linearized operator L_{E0} act on perturbations δE in the CUWF function space. The CUWF pseudo-spectrum under the metric-dependent operator norm $\|\cdot\|_{g_E}$ is

$$\sigma_\varepsilon(L_{E0}) \equiv \{ \lambda \in \mathbb{C} : \|(L_{E0} - \lambda I)^{-1}\|_{g_E} > \varepsilon^{-1} \}.$$

Equivalently,

$$\lambda \in \sigma_\varepsilon(L_{E0}) \Leftrightarrow \|(L_{E0} - \lambda I)f\|_{g_E} < \varepsilon \text{ for some } f,\ \|f\|_{g_E} = 1.$$

The key differences from classical pseudo-spectrum are that the norm depends on the entropic metric, the adjoint depends on that metric, strong non-normality makes σ_ε far larger than the true spectrum, and perturbations in DOF-space can shift σ_ε even when configuration-space perturbations are small. Thus σ_ε is the physically meaningful stability object in CUWF.

$$\|f\|_{g_E}^2 = \int f^2 |g_E| \, dx \, dq.$$

O-6.3.2 Why Pseudo-Spectrum Determines Stability in CUWF

Because CUWF evolution is

$$\partial E/\partial \tau = -\mathcal{E}_E(E),$$

the linearized flow around E_0 is

$$\delta E(\tau) \approx e^{-\tau L_{E0}} \delta E(0).$$

For non-normal operators, the stability of $e^{-\tau L_{E0}}$ is not determined by the spectrum of L_{E0}. Instead, transient amplification satisfies

$$\|e^{-\tau L_{E0}}\|_{g_E} \approx \sup_{\lambda \in \sigma_\varepsilon(L_{E0})} e^{-\tau \mathrm{Re}(\lambda)}.$$

If σ_ε enters the unstable half-plane, where Re $\lambda \geq 0$, the system can become unstable even when all true eigenvalues satisfy Re $\lambda < 0$. This is why CUWF tunneling can occur even when classical spectral analysis predicts stability.

O-6.3.3 Physical Meaning of Pseudo-Spectral Growth

Let $\tilde{\lambda} \in \sigma_\varepsilon(L_{E0})$ with $\mathrm{Re}(\tilde{\lambda}) \geq 0$. This implies non-normal amplification, collapse-slope competition, DOF-drift resonance, and hidden entropic channels. Small δE can trigger disproportionate geometric deformation; gradient sharpening may overpower curvature smoothing; cross-fiber coupling can create near-resonant amplification; and the entropic landscape can appear stable while operator geometry contains a soft direction enabling escape. These effects correspond to pre-tunneling zones, metastable collapse regions, and entanglement onset in multi-node CUWF systems.

O-6.3.4 Geometry of the σ_ε Boundary

The boundary $\partial\sigma_\varepsilon(L_{E0})$ satisfies

$$\|(L_{E0} - \lambda I)^{-1}\|_{g_E} = \varepsilon^{-1}.$$

This boundary identifies points where the operator almost fails to be invertible. In CUWF geometry, these are locations of maximal sensitivity, regions where metric-induced non-normality nearly destabilizes the flow, and thresholds for tunneling or instability. This boundary defines the quantum-classical crossover region.

O-6.3.5 Operator Criterion for CUWF Pseudo-Spectral Stability

A background field E_0 is pseudo-spectrally stable if and only if

$$\mathrm{Re}(\sigma_\varepsilon(L_{E0})) < 0 \text{ for all sufficiently small } \varepsilon.$$

If σ_ε touches Re $\lambda = 0$, the system reaches a critical metametric state. If σ_ε crosses into Re $\lambda > 0$, pseudo-spectral instability appears as tunneling or metastability. This matches the CUWF interpretation: stability corresponds to entropic convexity, instability to entropic concavity, and metastability to an entropic flat valley with DOF drift.

O-6.3.6 Relationship to Quantum–Classical Boundary

In CUWF, classical behavior is controlled by the true spectrum, whereas quantum or tunneling behavior is controlled by the pseudo-spectrum. Thus:

$$\text{Quantum regime} \Leftrightarrow \exists\, \tilde{\lambda} \in \sigma\_\varepsilon(L\_\{E0\}),\ \mathrm{Re}(\tilde{\lambda}) \geq 0.$$

$$\text{Classical regime} \Leftrightarrow \mathrm{Re}(\sigma\_\varepsilon(L\_\{E0\})) < 0.$$

This converts the vague idea of quantum behavior into a precise operator-theoretic condition.

O-6.3.7 Summary of CUWF Pseudo-Spectral Stability

The CUWF pseudo-spectrum is metric-dependent, detects hidden instability, governs tunneling thresholds, predicts entanglement onset, and generalizes classical stability to non-self-adjoint nonlinear operator systems. Pseudo-spectral stability is therefore more fundamental in CUWF than the classical spectrum.

O-6.4 Quantum–Classical Boundary in CUWF

The transition between classical and quantum behavior in CUWF is not defined by energy scale, decoherence threshold, or probabilistic postulate. Instead, it is defined entirely through operator geometry, using the pseudo-spectral structure of the linearized entropic-flow operator

$$L\_\{E0\} = (d/d\varepsilon)|\_\{\varepsilon=0\}\ \mathcal{E}\_E[E_0 + \varepsilon\delta E].$$

This section establishes the precise mathematical condition for the quantum-classical boundary and explains its geometric meaning in the full CUWF manifold.

O-6.4.1 Classical vs Quantum Regimes: Operator Definition

In classical physics, stability and evolution are dictated by the ordinary spectrum $\sigma(L_{E0})$. In CUWF, due to non-normality, metric dependence, DOF-fiber coupling, and curvature-induced mixing, the true determinant of physical behavior is the pseudo-spectrum $\sigma_\varepsilon(L_{E0})$.

A background field E_0 behaves classically if

$$\mathrm{Re}(\sigma_\varepsilon(L_{E0})) < 0 \text{ for all sufficiently small } \varepsilon.$$

This means all pseudo-spectral branches remain in the stable half-plane, no transient amplification occurs, flow follows smooth entropic descent, geometric deformations remain convex, and no tunneling channels are available. Classicality is therefore global entropic convexity.

Quantum behavior emerges precisely when

$$\exists\, \tilde{\lambda} \in \sigma_\varepsilon(L_{E0}) \text{ such that } \mathrm{Re}(\tilde{\lambda}) \geq 0.$$

A pseudo-eigenvalue moves into the unstable half-plane, transient amplification becomes possible, the operator becomes almost non-invertible in soft directions, hidden channels appear in the entropic geometry, and tunneling becomes allowed without violating deterministic flow.

O-6.4.2 Why "Quantum" = Pseudo-Spectral Activation

Quantum behavior in CUWF is not probabilistic. It is geometric activation of directions where L_{E0} is nearly singular. If $\tilde{\lambda} \in \sigma_\varepsilon(L_{E0})$ with $\mathrm{Re}(\tilde{\lambda}) \approx 0$, then small perturbations δE can produce large excursions before re-stabilizing. This is quantum-like because small seeds produce large geometric motion, metastable plateaus arise, tunneling corridors form through entropic concavity, bifurcation-like shifts occur in collapse-node geometry, and entanglement begins as DOF connections resonate.

In short:

Quantum = local non-normal amplification inside a globally stable geometry.

O-6.4.3 Entropic Geometry Interpretation

The pseudo-spectrum $\sigma_\varepsilon(L_{E0})$ depends on the local curvature of the entropic landscape, slope-curvature competition, cross-fiber DOF coupling, and metric gradient ∇g_E induced by E. Quantum behavior corresponds to regions where local concavity appears inside globally convex geometry. Equivalently, the entropic manifold folds, producing thin isthmus-like channels through which perturbations travel with unexpectedly low cost. These directions correspond to pseudo-eigenvectors associated with $\tilde{\lambda} \in \sigma_\varepsilon$.

O-6.4.4 Quantum–Classical Boundary as a Moving Interface

Unlike standard physics, where the quantum-classical transition is typically tied to fixed scales, in CUWF the boundary moves with E itself. The pseudo-spectrum depends on g_E, and g_E depends on E. Therefore, as E evolves,

$$\sigma_\varepsilon(L_{E0}) \text{ deforms continuously in time.}$$

Quantum zones can appear or disappear dynamically; classical regions can become quantum-active under curvature distortion; entanglement onset corresponds to merging pseudo-spectral branches; and collapse dynamics become history-dependent through metric evolution.

O-6.4.5 The Precise Boundary Condition

The CUWF quantum-classical boundary is the curve in the complex plane where

$$\mathrm{Re}(\sigma_\varepsilon(L_{E0})) = 0.$$

This boundary is a level set of the resolvent norm, marks the onset of near-singularity, identifies the transition from convex to semi-convex entropic geometry, functions as the operator equivalent of the WKB action barrier, generalizes the classical potential barrier, and determines tunneling without probabilistic postulates. Denote this boundary by

$$\partial QC(E_0) = \{ \lambda \in \mathbb{C} : \|(L_{E0} - \lambda I)^{-1}\|_{g_E} = \varepsilon^{-1} \}.$$

O-6.4.6 Physical Meaning of Crossing the Boundary

Crossing from the classical to the quantum regime corresponds to the emergence of a soft entropic direction, the appearance of a near-kernel for L_{E0}, breakdown of strictly convex flow, onset of non-normal geometric amplification, and opening of a tunneling corridor. This immediately triggers metastable behavior, tunneling, entanglement onset, and state branching in the sense developed in Papers A-4, A-5, and A-7.

O-6.4.7 Summary

The quantum-classical boundary in CUWF is not probabilistic or energy-based. It is a geometric boundary in operator space defined by $\sigma_\varepsilon(L_{E0})$. Classical behavior corresponds to entropic convexity and $\mathrm{Re}(\sigma_\varepsilon) < 0$. Quantum behavior corresponds to emergent concavity or soft directions and the existence of $\tilde{\lambda}$ with $\mathrm{Re}(\tilde{\lambda}) \geq 0$. This transforms the quantum regime into a precise operator-theoretic criterion.

O-6.5 Non-Normal Growth, Metastability, and Tunneling Thresholds in CUWF

The CUWF evolution equation

$$\partial E/\partial \tau = -\mathcal{E}_E(E)$$

is fully deterministic. Nevertheless, CUWF exhibits quantum-like transitions, metastability, and tunneling behavior. These are not introduced as axioms; they arise naturally from the non-normal structure of the linearized operator L_{E0} and its pseudo-spectral geometry $\sigma_\varepsilon(L_{E0})$.

O-6.5.1 Non-Normality as the Source of Transient Growth

A linear operator L is non-normal if

$$L L^{\dagger} \neq L^{\dagger} L.$$

For CUWF, L_{E0} is generally non-normal: the adjoint depends on g_E, DOF-fiber components mix under curvature, non-commuting sub-blocks generate shear-like amplification, and collapse-slope and curvature terms fail to align eigenvectors. True eigenvalues may all lie in Re $\lambda < 0$, while

$$\|e^{-\tau L_{E0}}\|_{g_E}$$

can still grow dramatically due to rotated eigenbases, nearly parallel left and right eigenvectors, and pseudo-eigenvalues approaching the unstable half-plane. This mismatch is the mathematical core of quantum-like CUWF behavior.

O-6.5.2 Metastability as an Entropic Plateau

A field configuration E_0 is metastable if the classical spectrum indicates stability, Re $\sigma < 0$, but $\sigma_\varepsilon(L_{E0})$ contains $\tilde{\lambda}$ with $\mathrm{Re}(\tilde{\lambda}) \gtrsim 0$. The true flow returns to equilibrium, but large excursions occur first; entropic curvature becomes semi-flat along one direction; collapse nodes drift before re-settling; and DOF drift produces temporary geometric displacement. Geometrically,

metastability = entropic convexity except for one soft corridor.

This corresponds physically to false-vacuum regions, quantum soft modes, slow-roll dynamics, and pre-tunneling geometry in Papers A-4 and A-5.

O-6.5.3 Tunneling Threshold Defined by Pseudo-Spectral Crossing

CUWF tunneling occurs not through probabilistic barrier penetration but through activation of a soft entropic direction. The tunneling threshold is reached when

$$\exists\, \tilde{\lambda} \in \sigma_\varepsilon(L_{E0}) \text{ with } \mathrm{Re}(\tilde{\lambda}) = 0.$$

As soon as $\mathrm{Re}(\tilde{\lambda}) \geq 0$, amplification outruns local curvature, entropic flow becomes directionally concave, a collapse node can exit its current basin, a new attractor becomes reachable, and a geometry-defined corridor opens. This provides a complete deterministic explanation for tunneling.

O-6.5.4 Non-Normal Amplification as a Physical Mechanism

Transient amplification has four main physical components: collapse-slope sharpening, DOF-connection resonance, entropic metric deformation, and curvature-induced shear. Gradients steepen faster than curvature can smooth; cross-fiber coupling creates constructive resonance; g_E evolves, changing the adjoint and rotating the operator; and the wave manifold twists, amplifying perturbations like shear flow. Together these produce sudden lifting of collapse nodes, multi-step geometric hops, propagation of instability through DOF fibers, and emergence of entanglement corridors.

O-6.5.5 Metastability Duration from Resolvent Geometry

The metastable lifetime τ_{meta} depends on how close $\sigma_\varepsilon(L_{E0})$ approaches Re $\lambda = 0$. Define

$$\Delta = \min_{\tilde{\lambda} \in \sigma_\varepsilon} |\mathrm{Re}(\tilde{\lambda})|.$$

Then

$$\tau_{meta} \sim 1 / \Delta.$$

Small Δ produces a long metastable plateau; $\Delta \longrightarrow 0$ indicates near-critical resonance; $\Delta < 0$ indicates tunneling or multi-node transition; and Δ controls how quantum-like the behavior appears. This replaces probabilistic lifetime formulas with deterministic operator geometry.

O-6.5.6 Relationship Between Non-Normality, Metastability, and Entanglement

In multi-node CUWF systems, non-normality causes pseudo-eigenvectors to align across fibers, amplification to propagate from node to node, resonant subspaces to form, and correlated geometry to emerge. This produces deterministic entanglement: not probabilistic, not collapse-postulate dependent, and not measurement-dependent, but purely geometric. Entanglement occurs when

$$\sigma_\varepsilon(L_{E0}^{(1)}) \text{ and } \sigma_\varepsilon(L_{E0}^{(2)}) \text{ merge or approach.}$$

O-6.5.7 Summary

CUWF exhibits quantum-like phenomena through deterministic operator geometry. Non-normality generates transient amplification and hidden growth directions; metastability produces semi-flat entropic regions and pre-tunneling drift; tunneling begins at pseudo-spectral crossing of Re $\lambda = 0$; and entanglement emerges from resonant merging of pseudo-spectral branches.

O-6.6 CUWF Resolvent Geometry and the Entropic Sensitivity Tensor

The pseudo-spectrum $\sigma_\varepsilon(L_{E0})$ identifies where the resolvent $(L_{E0} - \lambda I)^{-1}$ becomes large. CUWF, however, requires not only the location of amplification but also the way entropic geometry distributes sensitivity across configuration space and DOF fibers. Section O-6.6 introduces the Entropic Sensitivity Tensor, or EST, a metric-operator construct that quantifies the anisotropic response of E to perturbations δE.

O-6.6.1 Definition of the CUWF Resolvent

The resolvent of the linearized operator L_{E0} under the entropic metric g_E is

$$R(\lambda; E_0) = (L_{E0} - \lambda I)^{-1},$$

defined whenever the inverse exists. The magnitude $\|R(\lambda; E_0)\|_{g_E}$ encodes local softness of the entropic manifold, near-singular geometric directions, proximity to tunneling activation, and sensitivity of collapse-node trajectories. The pseudo-spectrum is the level set

$$\sigma_\varepsilon(L_{E0}) = \{ \lambda \in \mathbb{C} : \|R(\lambda; E_0)\|_{g_E} > \varepsilon^{-1} \}.$$

Thus the shape of $\|R(\lambda)\|$ determines the quantum readiness of E_0.

O-6.6.2 From Resolvent to Geometry: Motivation for the EST

The scalar norm $\|R(\lambda)\|_{g_E}$ gives the amplitude of growth but not the direction responsible for that growth. CUWF requires a geometric refinement identifying which perturbations δE produce maximum amplification, how sensitivity aligns with DOF fibers, how curvature and slope interact to create soft corridors, and how entanglement channels form through operator coupling. Classical stability theory cannot provide this because it uses eigenvectors of L, while CUWF requires pseudo-eigenvectors of the resolvent. This motivates the Entropic Sensitivity Tensor.

O-6.6.3 Definition of the Entropic Sensitivity Tensor (EST)

Let $u \in \mathrm{Dom}(L_{E0})$ be a perturbation direction with $\|u\|_{g_E} = 1$. Define the Entropic Sensitivity Tensor as

$$S_{E0}(\lambda) = R(\lambda; E_0)^\dagger R(\lambda; E_0),$$

where † is the adjoint under the metric g_E:

$$\langle u, v\rangle_{g_E} = \int u\, v\, |g_E|\, dx\, dq.$$

$S_{E0}(\lambda)$ is positive semi-definite. Its eigenvalues describe squared amplification factors, its eigenvectors describe directions of maximal sensitivity, and it depends continuously on λ and on the evolving entropic metric. This tensor translates pseudo-spectrum into tangible geometry.

O-6.6.4 Most Sensitive Direction and Soft Entropic Modes

The maximal eigenvalue of $S_{E0}(\lambda)$ is

$$\mu_{max}(\lambda) = \sup_{u : \|u\|=1} \|R(\lambda)u\|^2_{g_E},$$

and the corresponding eigenvector $v_{soft}(\lambda)$ satisfies

$$S_{E0}(\lambda)v_{soft} = \mu_{max}\, v_{soft}.$$

The vector v_soft(λ) is the softest direction in the entropic geometry and the pseudo-eigenvector closest to making $L_{E0} - \lambda I$ singular. When $\mathrm{Re}(\tilde{\lambda}) \geq 0$ for some $\tilde{\lambda} \in \sigma_\varepsilon$, v_soft($\tilde{\lambda}$) becomes the tunneling direction. Thus

$$\text{Tunneling} = \text{motion along the dominant eigenvector of } S_{E0}(\tilde{\lambda}).$$

O-6.6.5 Entropic Sensitivity Flow Along DOF Fibers

Let q denote DOF coordinates and x configuration coordinates. The EST can be decomposed into blocks:

$$S = [\ S_{xx} \quad S_{xq}\ ;\ S_{qx} \quad S_{qq}\ ].$$

Here S_{xx} describes geometric soft modes in configuration space, S_{qq} describes DOF-drift sensitivity, and S_{xq} and S_{qx} describe cross-coupling channels. A key CUWF result is that entanglement onsets when S_{xq} becomes comparable in magnitude to S_{xx}. This matches O-6.4 and O-6.5: pseudo-spectral merging leads to cross-fiber resonance, correlated drift, and deterministic entanglement.

O-6.6.6 Resolvent Geometry and Tunneling Thresholds

The boundary $\partial\sigma_\varepsilon(L_{E0})$ is determined by

$$\|R(\lambda; E_0)\|_{g_E} = \varepsilon^{-1}.$$

Using $S_{E0}(\lambda)$, this becomes

$$\mu_{max}(\lambda) = \varepsilon^{-2}.$$

Thus the tunneling threshold corresponds to crossing the level set $\mu_{max} = \varepsilon^{-2}$, metastability corresponds to μ_{max} slightly below ε^{-2}, and classicality corresponds to μ_{max} well below ε^{-2}.

O-6.6.7 Evolution of the EST Under Entropic Flow

As E evolves, g_E, L_{E0}, $R(\lambda)$, and $S_{E0}(\lambda)$ also evolve. This produces a sensitivity flow

$$\tau \longrightarrow S_{E(\tau)}(\lambda(\tau)).$$

Soft directions rotate over time, tunneling channels appear or disappear dynamically, DOF resonances drift to enable time-dependent entanglement, and classical regions can temporarily behave quantum-like. This explains intermittent quantumness in CUWF dynamical simulations.

O-6.6.8 Summary

The Entropic Sensitivity Tensor $S_{E0}(\lambda)$ refines pseudo-spectrum into directional geometric data, identifies the softest directions in the entropic manifold, determines tunneling paths, predicts metastability duration, encodes entanglement onset, and evolves dynamically under entropic flow. Resolvent geometry and EST together provide a deterministic explanation for quantum-like behavior in CUWF.

O-6.7 The CUWF Stability Cone and Entropic Curvature Criterion

Sections O-6.3–O-6.6 established pseudo-spectrum, resolvent geometry, and the EST. Section O-6.7 connects these to the CUWF Stability Cone, a global geometric concept that provides a curvature-based criterion for stability and determines the onset of quantum behavior.

O-6.7.1 Motivation: Connecting Operator Growth to Curvature

The core CUWF evolution depends on the structure of E and its induced metric g_E:

$$\partial E/\partial \tau = -\mathcal{E}_E(E).$$

Instability or tunneling occurs when curvature, slope, and coupling combine to create local concavity in some direction. Operator-theoretically, this appears as $\sigma_\varepsilon(L_{E0})$ entering $\mathrm{Re}\,\lambda \geq 0$, $\|R(\lambda)\|$

becoming large, and $\mu\_max(\lambda)$ increasing. Geometrically, entropic curvature becomes negative along some direction, collapse nodes fall into soft directions, and tunneling corridors open. CUWF formalizes this through the Stability Cone.

O-6.7.2 Definition of the CUWF Stability Cone

Let v be a perturbation direction with $\|v\|\_\{g\_E\} = 1$. Define the directional entropic curvature

$$K\_E(v) = v^T(\nabla^2 E)v,$$

where ∇^2E is the entropic Hessian under metric g_E. The sign of K_E(v) determines regime structure: K_E(v) > 0 means local convexity and classical stability, K_E(v) = 0 means soft plateau and metastability, and K_E(v) < 0 means local concavity and quantum activation. Define

$$C\_stable(E_0) = \{ v : K\_E(v) > 0 \},$$

$$C\_soft(E_0) = \{ v : K\_E(v) = 0 \},$$

$$C\_quantum(E_0) = \{ v : K\_E(v) < 0 \}.$$

These three cones partition the tangent space of the entropic manifold.

O-6.7.3 Relationship Between the Stability Cone and the Pseudo-Spectrum

The key CUWF result is that a direction v belongs to $C\_quantum(E_0)$ if and only if it aligns with a soft mode of $S\_\{E0\}(\tilde{\lambda})$ for some $\tilde{\lambda} \in \sigma\_\varepsilon$ with $Re(\tilde{\lambda}) \geq 0$. Equivalently,

$$v \parallel v\_soft(\tilde{\lambda}),$$

where v_soft is the eigenvector of $S\_\{E0\}(\tilde{\lambda})$ with maximal eigenvalue. Thus operator theory provides $\tilde{\lambda}$ and v_soft, while geometry provides the curvature K_E(v). The equivalence is

$$K\_E(v\_soft) < 0 \Leftrightarrow Re(\tilde{\lambda}) \geq 0.$$

O-6.7.4 Entropic Concavity as the Operator-Theoretic Quantum Trigger

Quantum activation occurs when

$$\exists\ v : K_E(v) < 0,$$

or equivalently, when

$$C_{quantum}(E_0) \neq \emptyset.$$

This matches the operator condition

$$\mathrm{Re}(\sigma_\varepsilon(L_{E0})) \geq 0.$$

At that point, entropic curvature becomes negative, flow accelerates along the soft direction, tunneling becomes possible, resonant DOF drift can begin, and entanglement channels can activate. The Stability Cone therefore gives a geometric interpretation of σ_ε-crossing.

O-6.7.5 Visualization of the Stability Cone

In local coordinates, the classical region corresponds to a strictly convex entropic surface, the metastable region to saddle-like flattening, and the quantum region to a strictly concave slice. As an analogy, the Stability Cone is the region above an upward paraboloid, the Soft Cone is the tangential flat plane, and the Quantum Cone is the region below a downward paraboloid. This provides an intuitive visualization of CUWF regime transitions.

O-6.7.6 Dynamics of the Stability Cone Under Entropic Flow

As E evolves, $\nabla^2 E$ changes shape, the cones C_stable, C_soft, and C_quantum deform, directions can move from stable to soft to quantum cones, σ_ε moves accordingly, and EST eigenvectors rotate to follow v_soft. This dynamic behavior explains intermittent tunneling, metastable drift, time-dependent entanglement, and oscillating quantum-classical transitions in CUWF simulations.

O-6.7.7 Curvature-Based Tunneling Threshold

The tunneling threshold has a precise curvature criterion:

$$\min_{v : \|v\|=1} K_E(v) = 0.$$

When the minimum directional curvature reaches zero, C_quantum first appears and $\sigma_\varepsilon(L_{E0})$ touches the imaginary axis. If

$$\min K_E(v) < 0,$$

then tunneling has activated, the EST identifies the tunneling direction, and σ_ε crosses into Re $\lambda \geq 0$. This is the geometric threshold equivalent to O-6.5 and O-6.6.

O-6.7.8 Summary

The Stability Cone framework establishes operator theory as equivalent to entropic curvature, pseudo-spectrum as equivalent to curvature sign, EST soft modes as equivalent to concave entropic directions, quantum regime as negative entropic curvature, metastability as zero entropic curvature, and classical regime as positive entropic curvature. O-6.7 therefore integrates the previous sections into one geometric object.

O-6.8 Multi-Node Stability and Collective Eigenmodes

Section O-6.8 generalizes the CUWF stability framework from single-node entropic fields to multi-node interacting systems on the CUWF hybrid manifold

$$\mathcal{M}_n = (C \times M\_DOF)^n.$$

Every collapse node perturbs the entropic metric, induces curvature deformations, and creates DOF-fiber connections that affect all other nodes. Thus the correct stability operator is block-structured, and the true spectrum is the pseudo-spectrum of the collective operator on $\mathcal{M}_n$. This section builds the mathematical foundation for deterministic entanglement, multi-node tunneling, collective soft-mode formation, collapse-node synchronization, and network-level instability.

O-6.8.1 Block Hessian on $\mathcal{M}_n$

Let the entropic field for n collapse nodes be

$$E = (E^{(1)}, E^{(2)}, \ldots, E^{(n)}).$$

The second-variation operator on the multi-node manifold is the block Hessian

$$H(E) = \begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1n} \\ H_{21} & H_{22} & \cdots & H_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ H_{n1} & H_{n2} & \cdots & H_{nn} \end{bmatrix}.$$

Here H_{ii} is the on-node curvature plus slope terms and DOF internal Hessian, while H_{ij} with $i \neq j$ is cross-node coupling curvature, DOF-mediated interaction, and metric-induced deformation. Importantly, $H_{ij} \neq 0$ whenever waves from node i propagate into the metric of node j. The block Hessian therefore reflects the true interacting geometry of CUWF: no collapse node evolves independently except in trivial or artificial limits.

O-6.8.2 On-Node, Cross-Node, and Collective Eigenmodes

The eigenmodes of the block operator fall into three major classes. On-node modes have most of their support on a single node and correspond to local curvature, local slope deformation, and node-specific DOF distortion; they reproduce the single-node stability analysis of O-6.3–O-6.7. Cross-node coupling modes have significant support on two or more nodes and arise through $H_{ij} + H_{ji}$ coupling, shared curvature deformation, metric resonance, and DOF-fiber cross-alignment; these modes are the mathematical basis of deterministic entanglement in CUWF. Collective eigenmodes are global modes with support across the full node set. They form when soft directions merge, curvature minima align, resolvent growth couples nodes into one mode, and pseudo-spectral branches converge. These explain collective tunneling, network-wide instability, synchronized collapse behavior, and multi-node entanglement patterns.

O-6.8.3 Collective Stability Criterion

For a single node, stability requires

$$\mathrm{Re}(\sigma\_\varepsilon(L\_\{E0\})) < 0.$$

For n nodes, the correct criterion is

$$\mathrm{Re}(\sigma\_\varepsilon(\mathcal{L}\_\{E0\})) < 0,$$

where $\mathcal{L}$ is the block linearized operator associated with the block Hessian $\mathcal{H}$. If even one eigenvalue or pseudo-eigenvalue of the block operator enters Re $\lambda \geq 0$, the entire multi-node system becomes collectively unstable. This explains how entanglement and tunneling propagate between nodes even when each individual node appears stable in isolation.

O-6.8.4 Operator Entanglement Eigenmodes

A deterministic entanglement mode in CUWF is an eigenvector of the block Hessian or block resolvent with non-zero components across multiple nodes and associated with a pseudo-eigenvalue near the stability boundary Re $\lambda \approx 0$. Mathematically:

$$H\ v\_ent = \lambda\_soft\ v\_ent, \quad v\_ent \neq \text{localized},$$

where λ_soft is close to zero or slightly negative. This mechanism requires no probabilistic interpretation, no Hilbert-space tensor product, and no measurement postulates. Correlation emerges as geometry of soft curvature directions.

O-6.8.5 Cross-Node Pseudo-Spectrum and Instability Spread

The multi-node pseudo-spectrum is

$$\sigma\_\varepsilon(\mathcal{L}) = \{ \lambda : \|(\mathcal{L} - \lambda I)\hat{}\{-1\}\|\_\{g\_E\} > \varepsilon\hat{}\{-1\} \}.$$

It is always larger than the union of single-node pseudo-spectra because coupling expands the resolvent norm. A soft mode in node i induces soft responses in node j, cross-node pseudo-eigenvectors propagate instability signatures, and instability spreads through the collective soft cone. This geometric structure carries tunneling influence, collapse drift, DOF-based correlation, and long-range entanglement.

O-6.8.6 Collective Tunneling and Soft-Mode Activation

In multi-node CUWF, tunneling is not activated at the individual node level; it activates when the collective curvature minimum becomes zero or negative:

$$\lambda\_\mathrm{min}(H) = 0,$$

or equivalently,

$$\mathrm{min}\_\{v \in T\mathcal{M}_\mathbf{n}, \|v\|=1\}\ v^T H v = 0.$$

A network-wide tunneling corridor then opens, collapse nodes drift in correlated fashion, pseudo-spectral branches cross Re $\lambda \geq 0$ collectively, entanglement modes become dominant, and local behavior becomes globally synchronized. This explains multi-particle tunneling, chain-reaction collapse events, entanglement propagation, and nonlocal quantum-like correlations without violating determinism.

O-6.8.7 Summary

Section O-6.8 completes the CUWF stability framework by incorporating multi-node interactions. The block Hessian governs global stability; on-node, cross-node, and collective eigenmodes describe geometry; the pseudo-spectrum of the block operator determines true stability; entanglement appears as a collective soft mode; instability and tunneling spread through resolvent-based coupling; and the stability of the CUWF universe is inherently a network property rather than a local one. This prepares Sections O-7 and O-8, where fully nonlinear multi-node dynamics and network bifurcations are developed.

Section O-6 — Conclusion

The analysis developed in Section O-6 establishes the first complete stability theory for CUWF dynamics on the hybrid manifold C × M_DOF, unifying operator theory, entropic geometry, and multi-node interactions into a single mathematical framework. The results can be summarized along four axes: stability structure, spectral behavior, geometric interpretation, and multi-node collective behavior.

1. What We Have Achieved: A Full Stability Framework for CUWF

Section O-6 formalizes the CUWF stability landscape using the metric-weighted Hessian, the CUWF pseudo-spectrum, adjoint-asymmetry and the quantum-classical boundary, the Entropic Sensitivity Tensor, and multi-node stability with collective modes.

The metric-weighted Hessian

$$\hat{S} = \mathrm{Hess}(E)$$

on the hybrid manifold with entropic metric g_E clarifies curvature modes, slope-driven modes, DOF-twist modes, positive/zero/negative eigenvalues, and how each contributes to classicality or quantum activation. It replaces the traditional quantum potential with a deterministic geometric structure.

The pseudo-spectrum

$$\sigma\_\varepsilon(L\_\{E0\})$$

captures non-normal amplification, collapse spikes, and transient instabilities—phenomena treated probabilistically in conventional quantum mechanics. It provides a deterministic criterion for tunneling activation and regime transition.

The Entropic Sensitivity Tensor

$$S_{E0}(\lambda) = R^\dagger R$$

adds directionality to stability theory by identifying the softest geometric direction, tunneling paths, entanglement channels, metastability durations, and regime-switching conditions. It is one of the most powerful mathematical tools introduced in this paper.

The extension of stability theory to the multi-node manifold M_n introduces block Hessians, block resolvents, cross-node coupling, network-wide eigenmodes, collective tunneling, and deterministic entanglement. This reveals that quantum correlations originate as curvature-aligned collective soft modes rather than probabilistic collapse.

2. What These Results Mean Physically

Positive curvature in all directions,

$$K_E(v) > 0,$$

corresponds to classical trajectories, stable collapse-node flow, absence of tunneling, negligible entropic drift, and absence of entanglement formation. Zero curvature,

$$K_E(v) = 0,$$

corresponds to flat entropic plateaus, metastability, increased sensitivity to perturbations, pseudo-spectral growth without full instability, and delayed tunneling effects. Negative curvature,

$$K_E(v) < 0,$$

corresponds to deterministic tunneling, activation of soft directions, DOF-drift coupling, entanglement mode formation, and nonlocal correlated collapse behavior. In CUWF, quantum behavior is not randomness but geometry-driven flow along negative-curvature directions.

Multi-node structures further show entanglement as geometric alignment, tunneling as network activation, cross-node instability spread, synchronized collapse events, and collective pseudo-spectral branching. These replace probabilistic many-body postulates with deterministic geometric foundations.

3. What We Can Use This For: Practical Applications in CUWF

O-6 equips CUWF with geometric, operator-theoretic, and directional criteria for predicting tunneling. It provides a deterministic mechanism for modeling entanglement through block eigenmodes, cross-node pseudo-spectrum, and collective soft-direction alignment. The block resolvent reveals how instability spreads in networks, enabling prediction of multi-particle tunneling, chain-reaction collapse, correlated DOF motion, and entropic-wave resonance. It also maps regime switching from classical to metastable to quantum, from stable to soft to unstable, and from isolated to cross-node to collective dynamics.

4. Why O-6 Is Foundational for the Rest of Paper C-3

The nonlinear and multi-node dynamics developed in later sections rely on curvature sign, pseudo-spectrum shape, EST eigenvectors, collective soft modes, and block-operator structure. Without O-6, the nonlinear stability analysis of O-7 and the network-bifurcation theory of O-8 would have no mathematical foundation.

Final Summary

Section O-6 builds the complete CUWF Stability Theory by unifying operator analysis, metric geometry, entropic curvature, pseudo-spectrum, and multi-node collective modes. This framework replaces the probabilistic assumptions of quantum mechanics with a deterministic, geometric, and physically intuitive structure based on entropic flow. It equips CUWF with robust mathematical tools to predict tunneling, entanglement, metastability, collapse-node synchronization, quantum-classical transitions, and network-level instabilities, all emerging naturally from the same fundamental entropic wave dynamics that define CUWF.