Section 5 — Topology Dynamics: PDE Conditions

In C-4 and C-5, the entropic manifold $\mathcal{M}^E$ was treated primarily at the level of geometry: local metric, curvature, geodesic motion, collapse basins, separatrices, conifold gates, and nonlocal adjacency. C-6 strengthens that picture. The PDE engine must not only evolve local fields; it must also encode when and how the topology of $\mathcal{M}^E$ changes.

The central principle of this section is straightforward:

The entropic manifold $\mathcal{M}^E$ is not static in topology.

Changes such as the birth of new basins, soft-mode bifurcations, conifold pinches, and wormhole connections must appear as intrinsic signatures of the PDEs governing fields on $\mathcal{M}^E$. They are not external manual rules added after the fact. They are threshold events generated by the same dynamical system that evolves stability, curvature, entanglement, and collapse flow.

This section formalizes the PDE conditions that signal and drive topology transitions:

birth of a new entropic basin (5.1);
soft-mode bifurcation of an existing basin (5.2);
conifold pinch formation through degeneration of the stability tensor (5.3);
wormhole transfer induced by strong entanglement (5.4);
a summary rule table for use in C-7 simulations (5.5);
boundary conditions and conserved or quasi-conserved functionals (5.6).

Throughout this section, τ denotes the entropic evolution variable, or collapse-time. The field $X^I(\sigma, \tau)$ denotes the configuration field on $\mathcal{M}^E$, where σ is an abstract label for initial conditions, configuration labels, or DOF indices. The entropic potential is written $\Phi(X, \tau)$. The tensor fields $g^{IJ}(\tau)$, $T^I{}_j(\tau)$, $\Xi^I{}_j(\tau)$, and $\mathcal{R}^I{}_j{}^{KL}(\tau)$ denote, respectively, the metric, stability tensor, entanglement tensor, and curvature tensor introduced earlier.

At a schematic level, metric evolution may be written as:

$$\partial g^{i}{}_{j} / \partial \tau = \mathcal{F}^{i}{}_{j}[g, \Phi, T, \Xi, \mathcal{R}]$$

for a tensorial functional $\mathcal{F}$ determined by the collapse-wave and curvature-evolution systems of Sections 3 and 4. Topology changes are not encoded by changing the form of $\mathcal{F}$. Instead, they occur when invariants built from these fields cross critical thresholds, such as curvature changing sign, eigenvalues of T vanishing, det(T) collapsing, or entanglement strength exceeding a critical bound.

5.1 PDE Condition for Basin Birth (K ⟶ +)

An entropic basin is a region of $\mathcal{M}^E$ in which collapse trajectories are attracted toward a local minimum of the entropic potential Φ. Geometrically, this corresponds to a patch in which curvature becomes locally positive and the potential Hessian becomes positive-definite.

Let $K(x, \tau)$ denote an appropriate scalar curvature associated with the region around a point x on $\mathcal{M}^E$. Depending on the effective dimension, K may represent a Gaussian curvature on a two-dimensional effective surface, or an averaged sectional curvature in higher-dimensional geometry.

Let $H(x, \tau)$ be the Hessian of the entropic potential:

$$H_{ij}(x, \tau) = \partial^2\Phi(x, \tau) / (\partial x_i \, \partial x_j)$$

and let $\lambda\_min(H)$ denote the smallest eigenvalue of H at (x, τ). A basin birth event at $x = x^*$ and $\tau = \tau\_c$ is defined by simultaneous curvature and stability formation.

First, curvature must cross into the positive regime:

$$K(x^*, \tau\_c) = 0, \quad \partial K/\partial \tau \, (x^*, \tau\_c) > 0$$

This means that the local geometry passes through zero curvature and becomes a positively curved well.

Second, local stability must form:

$$\lambda\_min(H(x^*, \tau\_c)) = 0, \quad \partial\lambda\_min(H)/\partial\tau\ (x^*, \tau\_c) > 0$$

This means that the potential landscape at x* gains a genuine local minimum as τ crosses τ_c.

A local basin indicator can therefore be defined by:

$$B(x, \tau) = 1 \text{ if } [K(x, \tau) > 0 \text{ and } H(x, \tau) \text{ is positive-definite}]$$

$$B(x, \tau) = 0 \text{ otherwise}$$

An elementary basin birth corresponds to the appearance of a new connected component of the set:

$$\{ x \mid B(x, \tau) = 1 \}$$

as τ passes through τ_c, where no such component existed for $\tau < \tau$_c.

Physically, this event means that the PDE evolution has reshaped the entropic manifold so that a new attractor region appears. Collapse trajectories entering this region approach a new classical state, such as a new macro-configuration, cosmological phase, or effective law-set. This rule allows basin birth to be detected numerically in C-7 without ad hoc intervention: the event is fully encoded in curvature evolution and potential shape.

5.2 Soft-Mode Bifurcation (λ_soft ⟶ 0)

While basin birth describes the appearance of a new attractor, soft-mode bifurcation describes the splitting of an existing basin into two or more branches. The key mechanism is the loss of stability along one eigen-direction of the stability tensor.

Recall that the stability tensor $T_{ij}(x, \tau)$ characterizes the local stability of collapse configuration. For definiteness, it may be identified approximately with the Hessian of Φ:

$$T_{ij}(x, \tau) \approx \partial^2\Phi(x, \tau) / (\partial x_i\, \partial x_j)$$

up to appropriate normalization factors, so that T and H carry the same eigenstructure.

Let $\{\lambda_\alpha(x, \tau)\}$ denote the eigenvalues of $T(x, \tau)$, and let λ_{min} be the smallest eigenvalue. Consider a basin centered at $x = x_0(\tau)$, where T remains positive-definite for $\tau < \tau_c$. A soft-mode bifurcation occurs if:

$$\lambda_{min}(x_0(\tau), \tau) \longrightarrow 0^+ \quad \text{as} \quad \tau \longrightarrow \tau_c^-$$

while all other eigenvalues remain positive and finite. At the critical point, the direction that was formerly stable becomes marginal. The instability condition is:

$$\partial\lambda_{min} / \partial\tau \, (x_0(\tau_c), \tau_c) < 0$$

In a local coordinate system aligned with the eigenbasis of T at x_0, let q denote the coordinate along the soft eigenvector v_{soft} corresponding to λ_{min}. Near the bifurcation, the local dynamics of q can be approximated by the normal-form equation:

$$\partial q / \partial\tau \approx \mu(\tau)q - \beta q^3 + \eta(\tau)$$

where $\mu(\tau_c) = 0$ and changes sign across τ_c, $\beta > 0$ is a stabilizing cubic term, and $\eta(\tau)$ represents an effective microscopic fluctuation or entropic noise term from the underlying background wave structure.

For $\tau < \tau_c$, the original basin remains stable. For $\tau > \tau_c$, the central fixed point becomes unstable and two new branches emerge. In CUWF interpretation, this is the mechanism by which a single entropic basin splits into multiple alternative collapse outcomes.

The term $\eta(\tau)$ does not introduce a fundamental probabilistic postulate. It represents microstructure in the entropic background that becomes amplified when λ_{soft} approaches zero. From the macroscopic viewpoint, this produces quantum-like branching: apparent randomness emerges from sensitivity at the bifurcation surface.

Numerically, C-7 must track the eigenvalues of T at basin centers, detect $\lambda_{min} \longrightarrow 0$ events, and realize branching by splitting the basin representation, for example by splitting graph nodes or duplicating attractor entries in a basin registry.

5.3 Conifold Pinch Formation (det T = 0)

Basin birth and soft-mode bifurcation describe topology changes in attractor structure. Conifold pinch formation is stronger: it describes the formation of a geometric neck where the stability tensor loses rank and the manifold develops a singular, pinched region.

Let det $T(x, \tau)$ be the determinant of the stability tensor at (x, τ). A conifold pinch at $x = x^*$ and $\tau = \tau_c$ is characterized first by rank deficiency:

$$\det T(x^*, \tau_c) = 0$$

At least one eigenvalue of T has reached zero, and the tensor no longer has full rank. The event becomes a true pinch when the determinant is driven downward by the PDE flow:

$$\partial(\det T) / \partial\tau \ (x^*, \tau_c) < 0$$

Locally, near the pinch, one can introduce coordinates (r, Ω) such that the metric takes the approximate form:

$$ds^2 \approx dr^2 + r^2 d\Omega^2$$

with $r(\tau) \longrightarrow 0$ as $\tau \longrightarrow \tau_c$. The cross-sectional radius of a neck in $\mathcal{M}^E$ therefore shrinks continuously to zero.

The consequence is that the manifold develops a conifold-like singularity where two regions are connected by a neck whose cross-section vanishes. Depending on the behavior of curvature $\mathcal{R}$ and entanglement Ξ, this neck may break into separated components or reconfigure into a wormhole-like bridge.

From the entropic viewpoint, a conifold pinch is a region where gradient flow becomes extremely focused, concentrating collapse trajectories into a narrow channel. It is a critical point of structural reorganization: smooth geometry breaks down, but CUWF still tracks the event through invariants such as det T, curvature, and entanglement strength.

In C-7, conifold detection will require monitoring det T(x, τ) and $\partial\tau$(det T), marking regions where the cross-section of the discretized manifold falls below a threshold, and triggering mesh or graph surgery according to the additional conditions in Sections 5.4 and 5.5.

5.4 Wormhole Transfer Regime ($\Xi_{eff} > \Xi_c$)

A conifold pinch is a local geometric degeneration. CUWF also incorporates the role of entanglement through Ξ, which can couple distant regions of $\mathcal{M}^E$. When a conifold pinch occurs while entanglement between two regions remains sufficiently strong, the resulting structure becomes a wormhole-like transfer channel for collapse.

Let $\Xi^I{}_j(\sigma, \sigma'; \tau)$ denote entanglement coupling between degrees of freedom labeled by σ and σ'. Define an effective entanglement strength between two regions A and B by:

$$\Xi_{eff}(A, B; \tau) = \int_A \int_B \Xi^I{}_j(\sigma, \sigma'; \tau)\, g^I{}_j(\sigma, \tau)\, dV(\sigma) dV(\sigma')$$

Regions A and B enter the wormhole transfer regime at $\tau = \tau_c$ when two conditions are met:

a conifold pinch geometrically connects A and B, forming a narrow neck as in Section 5.3;

the effective entanglement strength exceeds a critical value:

$$\Xi_{eff}(A, B; \tau_c) > \Xi_c$$

where Ξ_c is a model-dependent threshold determined by the PDE coefficients.

In this regime, the local collapse dynamics include a nonlocal transfer term. Schematically:

$$\partial X^I(\sigma, \tau)/\partial\tau = -\nabla^I\Phi(X(\sigma, \tau), \tau) - \int K^I{}_J(\sigma, \sigma'; \tau)\, \Xi^J{}_K(\sigma, \sigma'; \tau)[X^K(\sigma, \tau) - X^K(\sigma', \tau)]\, d\sigma'$$

Here $K^I{}_J$ is a kernel capturing the geometry of the neck and the local coupling structure. The integral is taken over σ' in region B, or more generally over all labels with significant $\Xi(\sigma, \sigma'; \tau)$.

The integral term enforces a synchronizing pull between configurations in A and B along strongly entangled directions. In the wormhole regime, this pull becomes dominant in the pinched neck, effectively identifying selected degrees of freedom across the two regions. Collapse in one region is mirrored in the other along the entanglement channel.

This gives a PDE-based account of nonlocal collapse. Correlations propagate through entanglement-structured necks in $\mathcal{M}^E$ rather than as signals through ordinary spacetime. The mechanism does not require superluminal signaling. Nonlocality is expressed as a topological property of the entropic manifold and its entanglement structure.

In C-7, wormhole detection and handling will require evaluating Ξ_{eff} between candidate regions, checking simultaneous pinch and entanglement threshold conditions, and modifying the discretized PDE through nonlocal adjacency or coupling matrices for the duration of the wormhole regime.

5.5 Topology Change Rule Summary Table

The previous subsections define distinct but related topology-change modes. For practical use in C-7, these conditions can be summarized as an event-detection rule set.

Mode	PDE Condition (Invariant Form)	Topological Effect	Physical Interpretation
Basin Birth	$K(x^*, \tau_c) = 0$, $\partial K/\partial\tau > 0$; $\lambda_{min}(H)$ becomes positive and $H \longrightarrow$ positive-definite	A new entropic basin appears as a connected region of positive curvature and stable Φ minimum	Emergence of a new classical attractor, macro-state, law-regime, or cosmological phase

Soft-Mode Bifurcation	λ_min(T($x_0(\tau)$, τ)) ⟶ 0; $\partial\lambda$_min/$\partial\tau$ < 0; local form $\partial q/\partial\tau \approx \mu q - \beta q^3 + \eta$	An existing basin splits into multiple branches along a soft direction	Quantum-like branching: a single attractor resolves into several possible outcomes
Conifold Pinch	det T(x*, τ_c) = 0; ∂(det T)/$\partial\tau$ < 0; neck radius r(τ) ⟶ 0	A geometric neck forms; the manifold may separate or reconnect	Critical restructuring: the manifold approaches split, flip, or reconnection
Wormhole Transfer	Conifold pinch + Ξ_eff(A, B; τ_c) > Ξ_c; nonlocal coupling term dominates	Selected DOFs in A and B become topologically linked through an effective bridge	Nonlocal collapse transfer: entangled regions behave as connected through a wormhole in $\mathcal{M}^E$

These rules are not external axioms. They are threshold conditions on invariants of the PDE system. C-6 provides the analytical formulation, while C-7 implements the event-detection layer that monitors K, eigenvalues of H and T, det T, and Ξ_eff to trigger basin creation, splitting, merging, separation, or wormhole-like linking.

5.6 Boundary Conditions and Conserved Functionals

Although Section 5 defines topology-change rules, the PDE engine must also specify boundary behavior on $\mathcal{M}^E$ and identify which global functionals are conserved, dissipated, or tracked under collapse evolution.

Because $\mathcal{M}^E$ is not a fixed-shape spatial container and can change topology, boundaries cannot always be treated as rigid spatial edges. Three admissible boundary modes are defined.

Absorbing boundary: trajectories reaching $\partial\mathcal{M}^E$ are removed into a sink; E decreases monotonically.

Reflecting boundary: entropic flow normal to $\partial\mathcal{M}^E$ vanishes. This is a no-flux boundary condition:

$$n\cdot\nabla\Phi = 0, \quad n\cdot J = 0$$

where J is the collapse current.

Periodic / identification boundary: $\partial\mathcal{M}^E$ is topologically glued. This mode is used when $\mathcal{M}^E$ is compactified or becomes multi-connected after a wormhole event.

In C-7, the boundary mode should be selectable per simulation regime. The default mode is periodic or identified, consistent with a boundary-less universe.

Collapse is globally dissipative rather than Hamiltonian. Nevertheless, several bulk quantities must be tracked.

Probability / occupancy conservation (global mass):

$$\int \psi(x, \tau)\, dV = \text{const}$$

unless absorbing boundary conditions or basin annihilation occur.

Volume of $\mathcal{M}^E$ (geometric measure):

not strictly conserved; it evolves under curvature flow because $\bar{R}(\tau)$ modifies the measure.

Collapse energy functional:

$$E\_collapse = \int \psi(x, \tau)\, \Phi(x, \tau)\, dV$$

which decreases monotonically under ideal collapse flow.

Thus CUWF behaves as a dissipative, attractor-forming PDE system rather than as a reversible Hamiltonian field theory. Entropy-weighted collapse drives the system toward basin minima unless topology transitions, conifold events, or wormhole connectivity disrupt the descent.

Section 5 Summary

Section 5 completes the bridge from local field evolution to global topology evolution on the entropic manifold. Basin birth, soft-mode bifurcation, conifold pinch formation, and wormhole transfer are all expressed as threshold events in the PDE fields. Boundary conditions and conserved or quasi-conserved functionals then define how the PDE system behaves globally under simulation.

The result is that CUWF can treat basin structure, quantum-like branching, conifold gates, and nonlocal collapse as consequences of one evolving PDE engine on $\mathcal{M}^E$.