Section 6. Multi-Scale Collapse Dynamics

Paper C-6: PDE Dynamics of the Entropic Manifold — Section 6

Sections 3–5 defined the local PDEs on the entropic manifold $\mathcal{M}^E$ and the conditions under which those equations drive topology change: basin birth, soft-mode bifurcation, conifold pinch formation, and wormhole transfer. Section 6 now extends that local and topological picture into a multi-scale dynamical architecture.

The observable universe is not tied to a single geometric scale. Collapse phenomena organize themselves through nested levels of structure: macro-scale curvature, meso-scale basins, and micro-scale sub-basins. The same PDE engine governs all levels, but different effective variables dominate after coarse-graining or refinement.

The hierarchy is organized as follows:

Macro-scale: global curvature and large-scale entropic flows determine cosmological structure and background law-sets.

Meso-scale: individual basins drift, merge, migrate, and annihilate, producing effective classical phases, quasi-particles, and domains.

Micro-scale: sub-basins and fine-grained collapse structures produce quantum spectra, metastable states, and detailed collapse statistics.

Let ℓ denote a characteristic scale on $\mathcal{M}^E$, measured using an intrinsic metric derived from $g^I{}_j$. For each ℓ, define a coarse-graining operator C_ℓ that maps the full configuration fields into effective fields at scale ℓ.

The multi-scale structure is then encoded by:

Macro-geometry: $\ell \gg$ typical basin size.

Meso-evolution: $\ell \approx$ basin size and inter-basin separation.

Micro-collapse hierarchy: $\ell \ll$ basin size, resolving internal sub-basin structure.

Renormalization-like flows of degrees of freedom across scales and self-similar attractor cascades complete the picture. This section therefore provides the scale architecture that C-7 will later translate into adaptive resolution, dynamic refinement, and solver-level coarse-graining.

6.1 Macro-Geometry (Global Curvature)

At the macro-scale, individual basins and local defects are not resolved. Instead, the entropic manifold is treated as a smoothed geometric object described by coarse-grained curvature fields and an averaged entropic potential. This level captures the large-scale shape of $\mathcal{M}^E$ and its cosmological regimes.

Define a coarse-grained metric $\bar{g}^I{}_j(\tau)$ and curvature tensor $\mathcal{R}^I{}_j{}^{KL}(\tau)$ by applying a smoothing operator at macro-scale ℓ_macro:

$$\bar{g}^I{}_j(\tau; \ell\_macro) = C\_\ell\_macro[g^I{}_j(\cdot, \tau)]$$

$$\mathcal{R}^I{}_j{}^{KL}(\tau; \ell\_macro) = C\_\ell\_macro[\mathcal{R}^I{}_j{}^{KL}(\cdot, \tau)]$$

From $\mathcal{R}$, one may construct a scalar macro-curvature $\bar{R}(\tau)$, such as the Ricci scalar of the coarse-grained metric. The macro-geometry may then be governed schematically by:

$$\partial \bar{g}^I{}_j / \partial \tau = -2\, \bar{R}^I{}_j + S^I{}_j[\bar{g}, \bar{T}, \bar{\Xi}]$$

Here $\bar{R}^I{}_j$ is the coarse-grained Ricci tensor, while $S^I{}_j$ collects contributions from embedded basins, residual large-scale entanglement, and long-wavelength modes of the entropic potential.

Macro feature	Meaning in CUWF
Macroscopic entropic flow	$\bar{R}(\tau)$ captures the large-scale shape of $\mathcal{M}^E$. Changes in $\bar{R}$ correspond to transitions between cosmological regimes.

Effective law-sets	Fine basin details are averaged into macro-level PDE parameters that determine which local dynamics are available at smaller scales.
Global attractors	Stable configurations of $(\bar{g}, \bar{R})$ under the macro-PDE correspond to long-lived cosmological epochs.

In C-7, macro-geometry will be represented through low-resolution discretization of $\mathcal{M}^E$, where individual basins are not resolved but global curvature evolution and entropic flux remain directly simulated.

6.2 Meso-Evolution (Basin Drift & Migration)

At the meso-scale, individual entropic basins are resolved as discrete structures embedded within the macro-geometry. Each basin behaves like a dynamical object whose position, depth, shape, and connectivity evolve under the same underlying PDE engine.

Each basin a is characterized by:

Center position $X_a(\tau)$ on $\mathcal{M}^E$.

Basin shape encoded by an effective Hessian or stability matrix $T_a(\tau)$.

Basin depth $\Delta\Phi_a(\tau)$, defined by the difference between the local minimum and surrounding saddle points.

Connectivity to neighboring basins through barrier heights, shared separatrices, or entanglement-mediated links.

The meso-dynamics can be approximated by projecting the full PDE onto localized basin modes. The basin center then drifts according to an effective potential:

$$dX_a / d\tau \approx -\nabla\Phi\_eff(X_a; \{other\ basins\}, \tau)$$

Here Φ_eff combines the coarse-grained entropic potential, interactions among basins, and entanglement-mediated couplings active at the meso-scale.

Basin shape and stability also evolve. Representative meso-scale flow equations are:

$$dT_a / d\tau = G(T_a, \bar{R}, \bar{\Xi}, \ldots)$$

$$d\Delta\Phi_a / d\tau = H(T_a, \bar{R}, \ldots)$$

The functionals G and H are effective evolution operators derived from averaging the full PDE over the basin region. They describe basin sharpening, basin flattening, growth or decay of entropic depth, and approach toward bifurcation or annihilation.

Meso event	Description
Migration	A basin center drifts through $\mathcal{M}^E$ as macro-curvature and effective potential gradients change.
Merging	Two basins fuse when an intermediate barrier vanishes.
Annihilation	A basin fades when $\Delta\Phi_a \to 0$ or when its volume collapses into neighboring basins.
Boundary alignment	Basin separatrices stabilize along macro-curvature ridges or valleys.

Phenomenologically, meso-evolution models classical phase transitions, domain migration, and the emergence of effective degrees of freedom such as quasi-particles or classical fields. C-7 will represent this level by tracking a finite basin registry with dynamically updated positions, shapes, depths, and connectivity.

6.3 Micro-Collapse Hierarchy (Sub-Basins)

At the micro-scale, a single meso-basin may contain a rich internal hierarchy of sub-basins. These structures correspond to fine-grained quantum states inside a classical macro-configuration, local minima separated by small entropic barriers, and metastable states with finite lifetimes under entropic evolution.

Let basin a occupy a region $\mathfrak{B}_a \subset \mathcal{M}^E$. Within $\mathfrak{B}_a$, restrict the entropic potential as:

$$\Phi_a(x, \tau) = \Phi(x, \tau), \quad x \in \mathfrak{B}_a$$

The micro-collapse hierarchy is obtained by decomposing Φ_a into local sub-basins. Each local minimum $x_{a,k}(\tau)$ defines a sub-basin (a,k) with position $x_{a,k}(\tau)$, local stability tensor $T_{a,k}(\tau)$, and depth $\Delta\Phi_{a,k}(\tau)$ relative to the surrounding micro-landscape.

Sub-basins are organized by barrier heights. Low barriers correspond to fast transitions and near-degenerate states. High but finite barriers correspond to metastable configurations with longer lifetimes. This generates a micro-transition graph whose nodes are sub-basins and whose edges encode transition accessibility derived from entropic gradients and barrier heights.

At micro-scale, the dynamics inside $\mathfrak{B}_a$ can be approximated by a reduced PDE:

$$\partial\psi_a(x, \tau)/\partial\tau = -\nabla\cdot(\psi_a \nabla\Phi_a) + \mathfrak{D}_a \nabla^2\psi_a + N_a[\psi_a, \Xi]$$

Here $\psi_a(x, \tau)$ is an occupancy or probability-like measure over micro-states, $\mathfrak{D}_a$ is an effective entropic diffusion coefficient, and N_a contains nonlinear and entanglement-driven terms.

The long-time behavior of this PDE is dominated by occupation of the deepest sub-basins, transitions among near-degenerate sub-basins, and quantum-like stationary distributions. In physical terms, the micro-collapse hierarchy provides CUWF's counterpart of fine-grained energy spectra, band structures, and state multiplicity inside an apparently single classical configuration.

6.4 Renormalization Flow of DOF Structure

The existence of macro-, meso-, and micro-scales implies that the effective number and type of degrees of freedom depend on the observation scale ℓ. Section 6.4 formalizes this using a renormalization flow of DOF structure on $\mathcal{M}^E$.

Let $N(\ell, \tau)$ denote the effective number of DOFs at scale ℓ and entropic time τ. At fine scale, N is large because microscopic collapse-node DOFs remain resolved. As ℓ increases, many micro-DOFs are integrated out and only collective modes remain.

This scale dependence can be encoded by a beta-function-like relation:

$$\partial N(\ell, \tau) / \partial \ln\ell = \beta_N(\ell, \tau; \{\text{fields}\})$$

Other effective quantities, such as coarse-grained coupling constants, diffusion coefficients, or entropic stiffness parameters, can evolve similarly:

$$\partial g_{eff} / \partial \ln\ell = \beta_g(g_{eff}, \Xi_{eff}, \ldots)$$

$$\partial \kappa_{eff} / \partial \ln\ell = \beta_\kappa(\kappa_{eff}, \ldots)$$

Fixed points of these flows, such as $\beta_N = 0$, $\beta_g = 0$, and $\beta_\kappa = 0$, correspond to self-consistent scale-invariant collapse regimes. Crossovers occur when the system moves from one fixed-point basin to another as τ evolves, or when macro-curvature changes alter the beta functions.

6.4.1 Scaling Law for Renormalized DOF Flow

For implementation in C-7, the renormalized DOF flow can be expressed as a direct update rule for an effective DOF count. Define:

$$N_{eff}(\tau + \Delta\tau) = R\{ N(\tau) \mid \lambda_{soft}(\tau), \mathcal{R}(\tau), \Xi_{eff}(\tau), \ldots \}$$

The qualitative dependencies are:

$\lambda_{soft} \downarrow \longrightarrow$ DOF compress.

$|\mathcal{R}| \uparrow \longrightarrow$ basin coarse-merging.

Ξ_eff ↑ ⟶ shared DOF between linked regions.

Topology events ⟶ discrete jumps in N_eff.

This gives C-7 a direct algorithmic rule for updating resolution levels as collapse proceeds. When stability softens, curvature intensifies, entanglement increases, or topology changes, the solver can adjust the effective DOF structure rather than treating resolution as externally fixed.

6.5 Self-Similar Attractor Cascades

The final component of multi-scale dynamics is the emergence of self-similar attractor cascades. These occur when attractors, sub-attractors, and their transition structures replicate related patterns across multiple scales.

A self-similar attractor cascade occurs when the basin hierarchy {basins, sub-basins, sub-sub-basins, …} exhibits repeated patterns under scale transformations, and when the statistics of basin sizes, depths, and barrier heights follow approximate scaling laws such as:

$$\text{Number of basins of size} \approx \ell \propto \ell^{(-D\_eff)}$$

From the PDE perspective, such cascades appear when the renormalization flow approaches a nontrivial fixed point with scale-invariant structure. The beta functions stabilize, and distributions of entropic gradients, curvature values, and stability tensors obey approximate scaling relations.

In this regime, collapse dynamics generate repeated cycles of basin birth, bifurcation, and pinch at progressively smaller scales. The result may include bursts of activity followed by quiescent intervals, with similar temporal statistics recurring across scales.

Physically, self-similar attractor cascades provide a CUWF-based explanation for hierarchical structure, scale-free statistics, intermittency, and the persistence of patterns across widely separated scales. These cascades are not separate postulates; they are consequences of the entropic PDE engine, the topology transition rules of Section 5, and the renormalization flow of DOFs and parameters.

C-7 can detect these cascades by monitoring basin and sub-basin statistics over multiple resolution levels, fitting scaling laws to basin size, depth, and lifetime distributions, and checking whether these statistics remain approximately invariant under scale change.

Summary of Section 6

Section 6 extends the C-6 PDE framework from local topology dynamics into a multi-scale description of the entropic manifold. It establishes:

Layer	Role
Macro-geometry	Tracks global curvature, cosmological-scale entropic flow, and effective law-sets.
Meso-evolution	Governs basin drift, migration, merging, annihilation, and phase-like behavior.
Micro-collapse hierarchy	Resolves sub-basin structures, fine-grained quantum-like states, and metastable transition graphs.
Renormalization flow	Describes how DOF structure and effective parameters evolve across scales.
Self-similar attractor cascades	Capture scale-invariant hierarchies of collapse structures and recurring transition patterns.

Together, Sections 5 and 6 complete the conceptual bridge from local PDE dynamics to global, multi-scale, and topological behavior of $\mathcal{M}^E$. This provides the necessary foundation for C-7 to implement adaptive refinement, basin registries, resolution updates, and full CUWF simulation across scales.